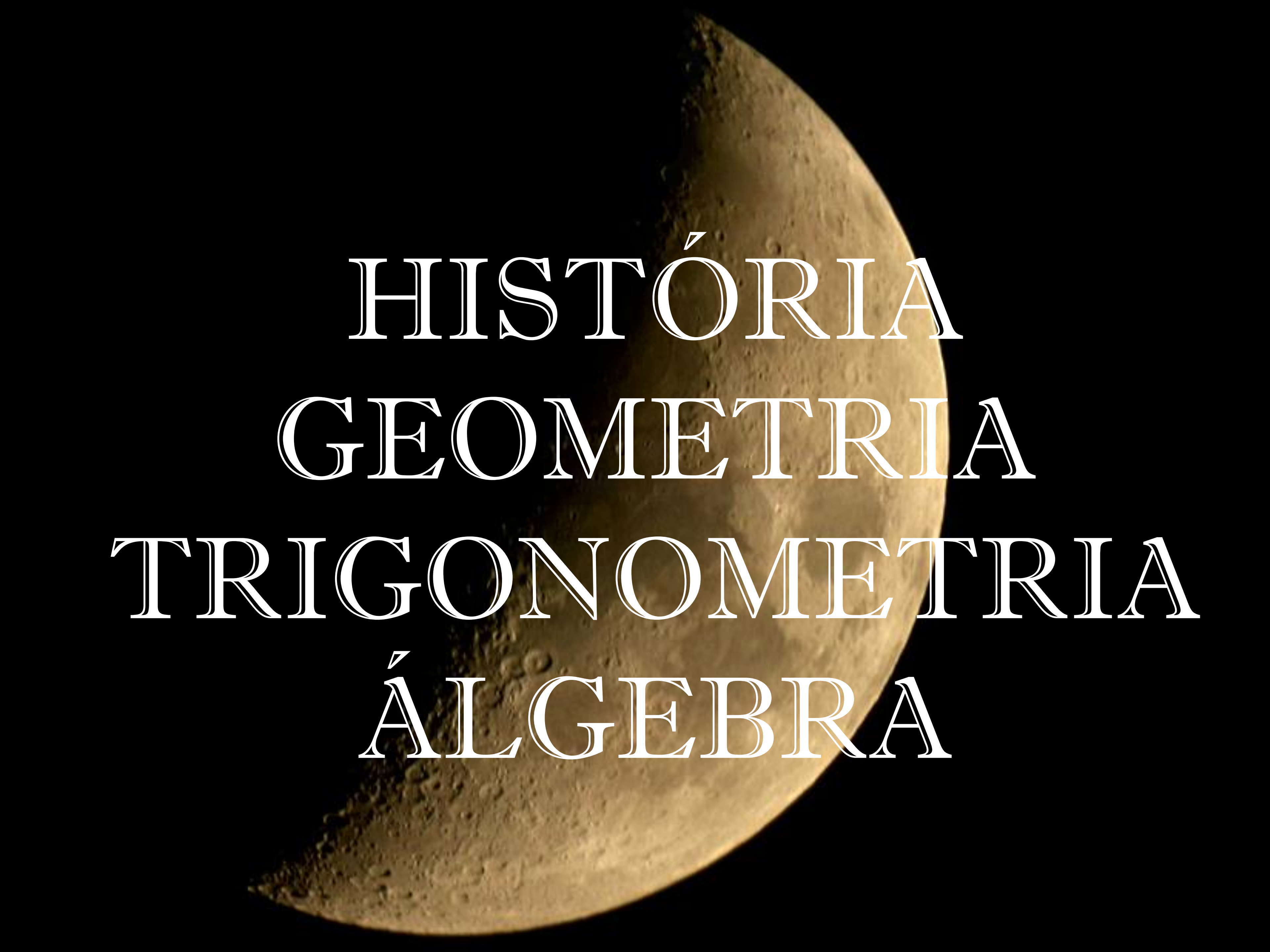
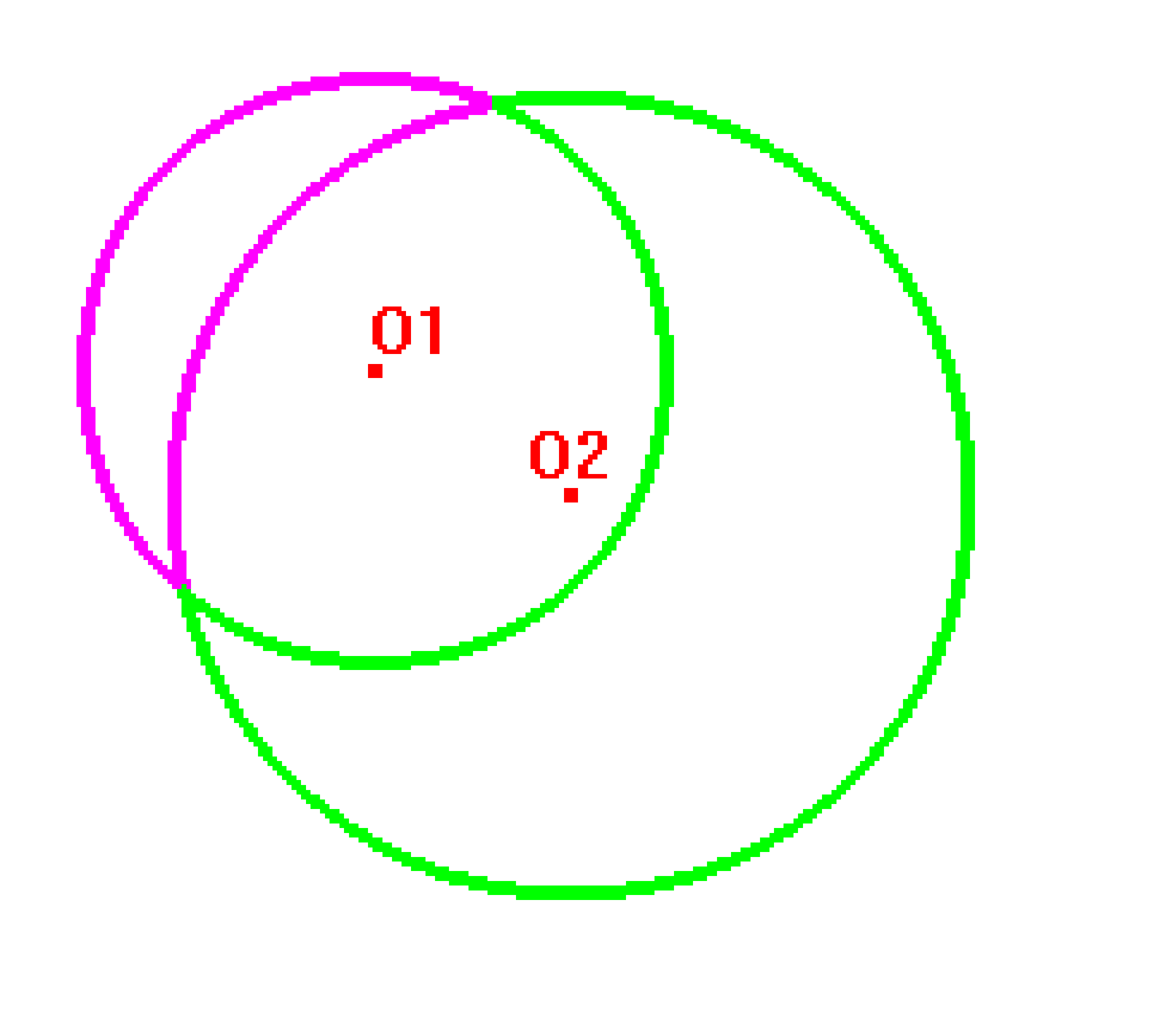
AS LUAS DE HIPÓCRATES

EHH – Novembro - 2018
M. Elisa E. L. Galvão
UNIAN/IME-USP


HISTÓRIA
GEOMETRIA
TRIGONOMETRIA
ÁLGEBRA

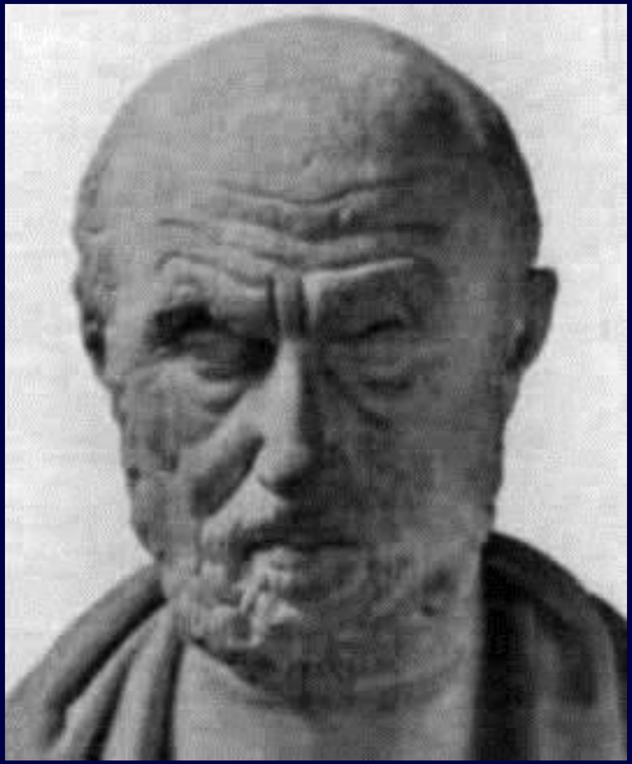
LUAS

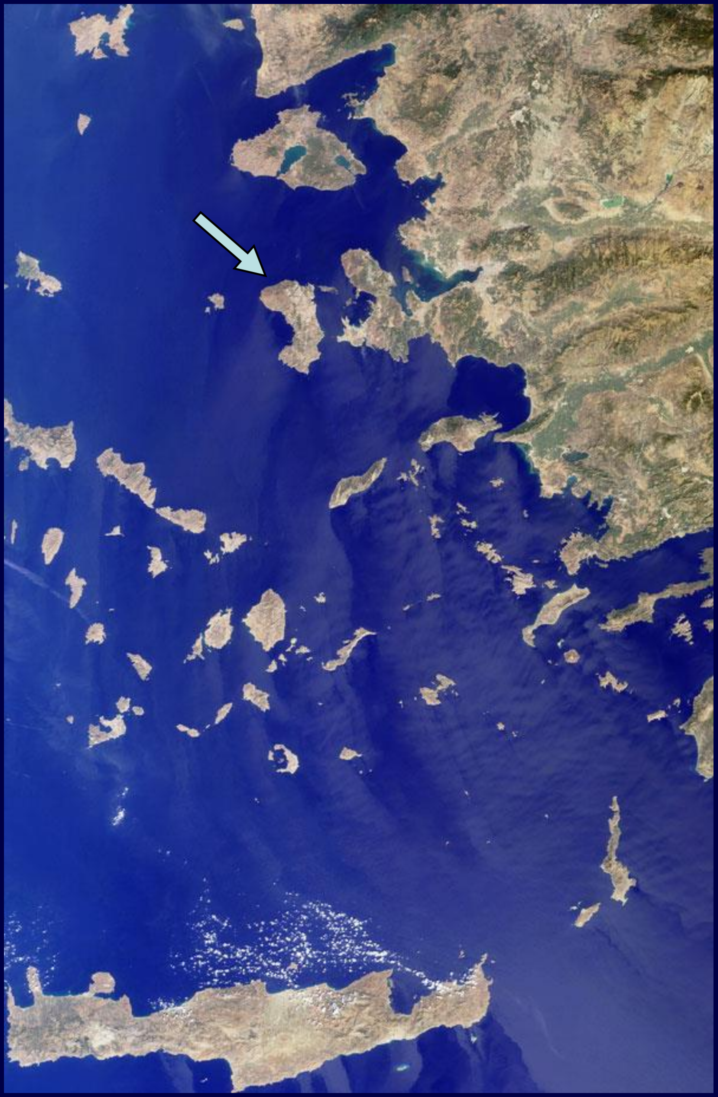


QUADRATURA

http://www.interarteonline.com/Sixto_Aguilera/Altas/Lunas.jpg

HIPÓCRATES DE CHIOS
Séc. V AC





Kavala
Alexandroupoli
Thasos
Thessaloniki
Samothraki
Limnos
Kerkyra
Igoumenitsa
Alonissos
Skiathos
Skopelos
Lesvos
Lefkada
Agh. Konstandinos
Skyros
Kimi
Euboea
Chios
Kefalonia
Patra
ATHENS
Rafina
Lavrion
Zakynthos
Andros
Samos
Ikaria
Tinos
Kea
Poros
Syros
Kythnos
Leros
Serifos
Paros
Kalymnos
Sifnos
Naxos
Gythion
Amorgos
Kos
Milos
Ios
Astipalea
Simi
Santorini
Kythira
Karpathos
GREECE
Chania
Rethymno
Kassos
Iraklio
Ag. Nikolaos

SOBRE HIPÓCRATES...

450 a.C / 430 a.C – Atenas

- Elementos de Geometria -

História da Geometria – Eudemian Summary
- Eudemus – Escola de Aristóteles

Comentários de Simplício – séc. VI

PROBLEMA

Podemos construir,
com régua e compasso,
um quadrado equivalente
a um círculo?

QUADRATURA DO CÍRCULO

QUADRATURA DE UMA FIGURA GEOMÉTRICA

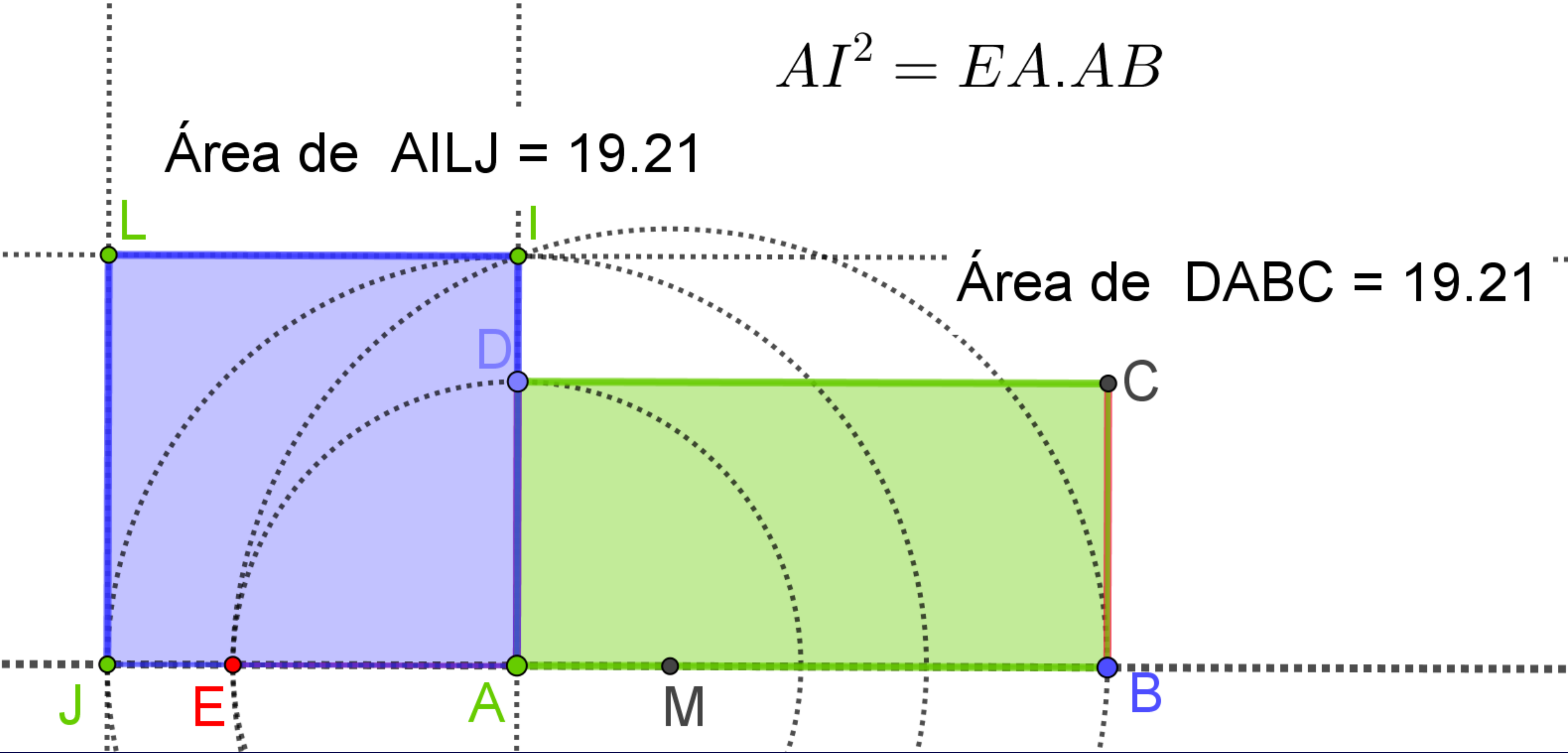
construção, com régua não graduada e compasso, de um quadrado equivalente à ela

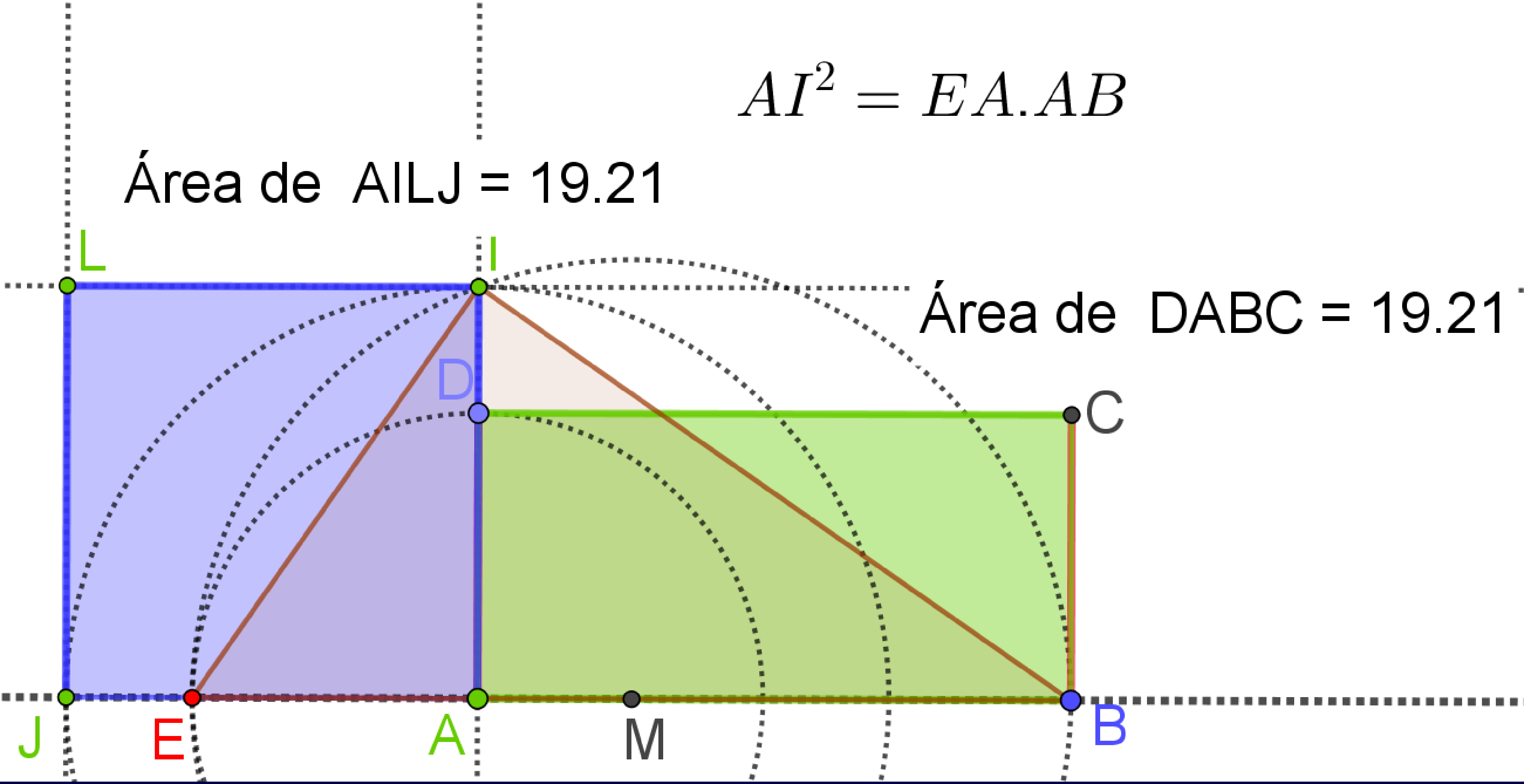
(equivalente: mesma área)

Oenópides de Chios
(490 – 420 a.C.)

primeiras construções com régua e compasso

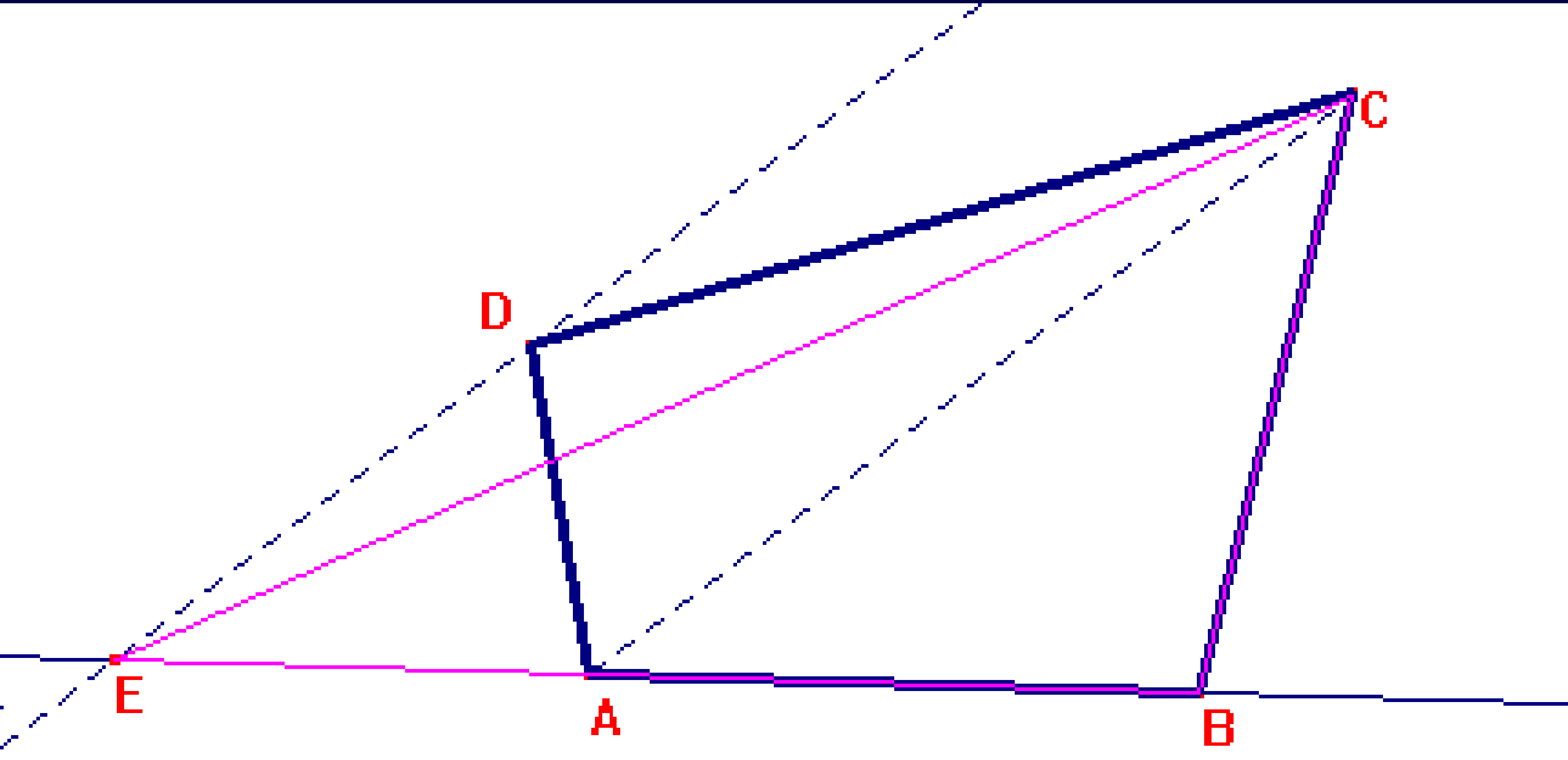
QUADRATURA DO RETÂNGULO





$AI^2 = EA.AB$
Área de AILJ = 19.21
Área de DABC = 19.21
L
I
D
C
J
E
A
M
B

QUADRATURA DE POLÍGONOS



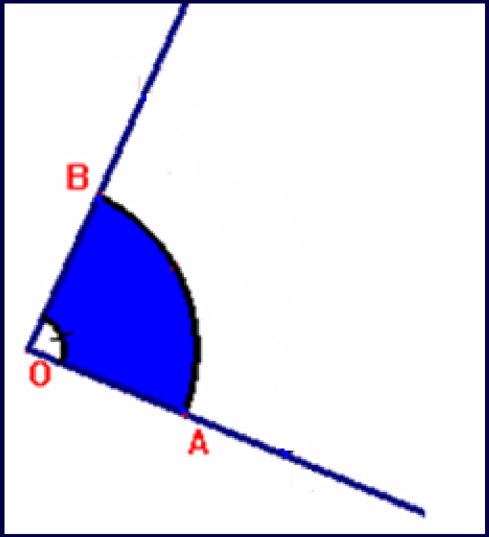
O TRABALHO DE HIPÓCRATES:

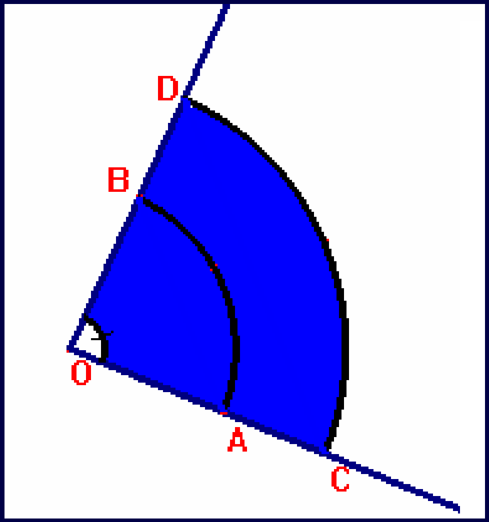
MÉTODO E EXEMPLOS

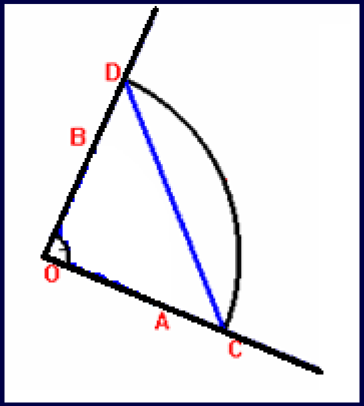
QUADRATURA DAS LUAS

Como construir, com régua e compasso, um quadrado equivalente a uma lua?

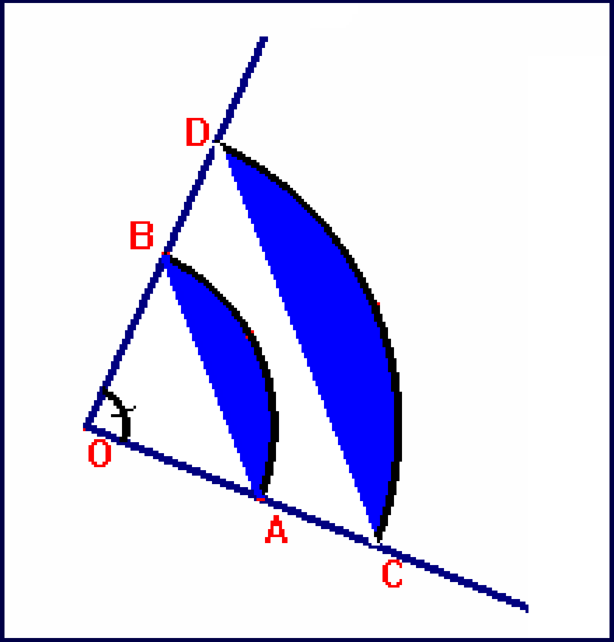
Setores Circulares - Semelhança



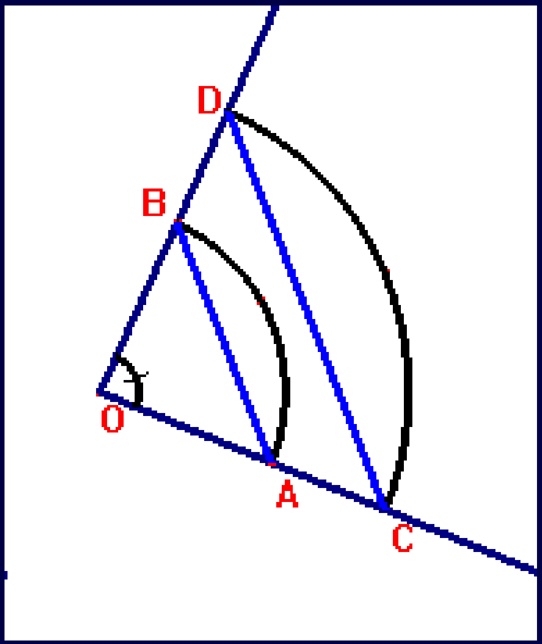




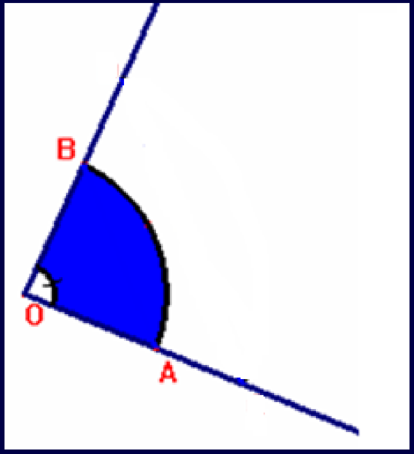
Segmentos Circulares

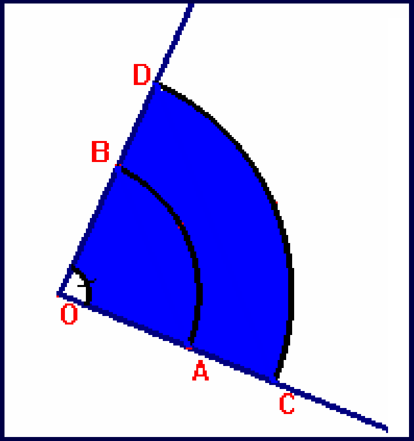


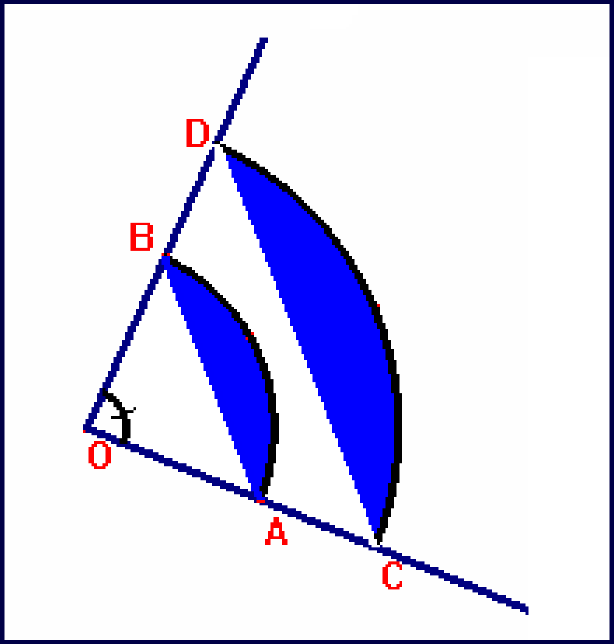
Áreas de Setores ou Segmentos Circulares



$$\frac{A_1}{A_2} = \frac{AB^2}{CD^2}$$

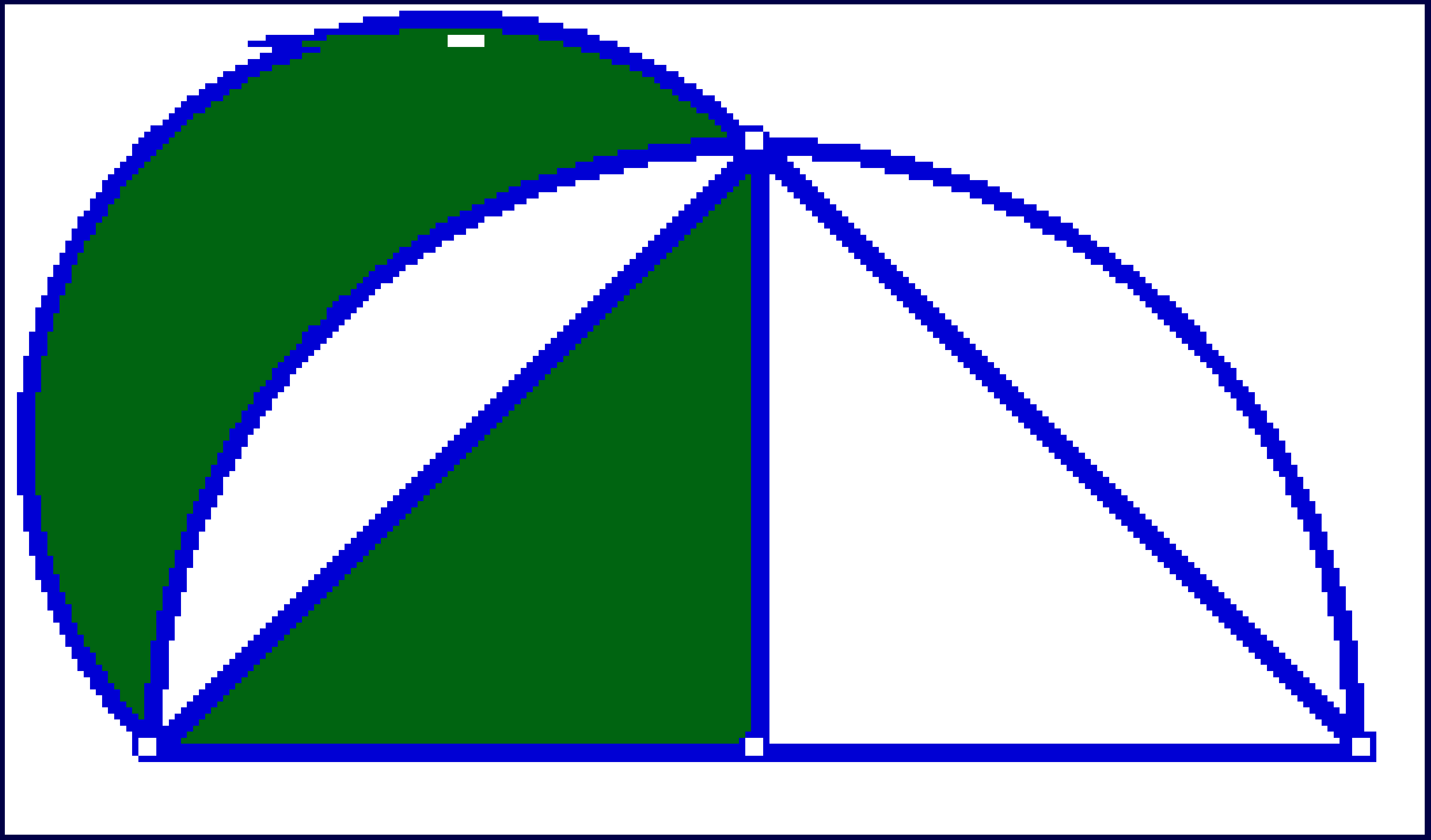


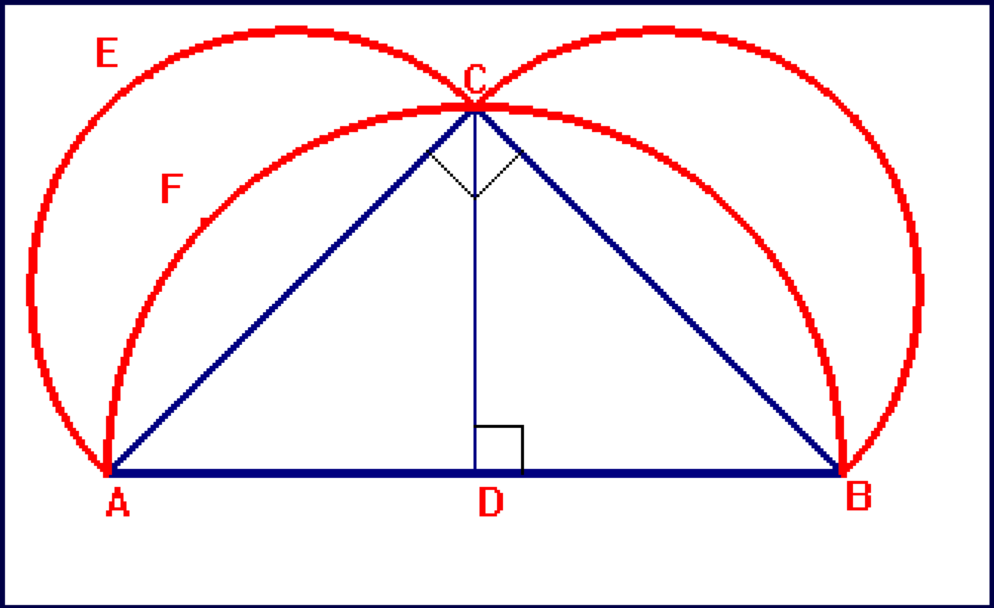




Primeira lua de Hipócrates:

a área da lua é igual à área do triângulo

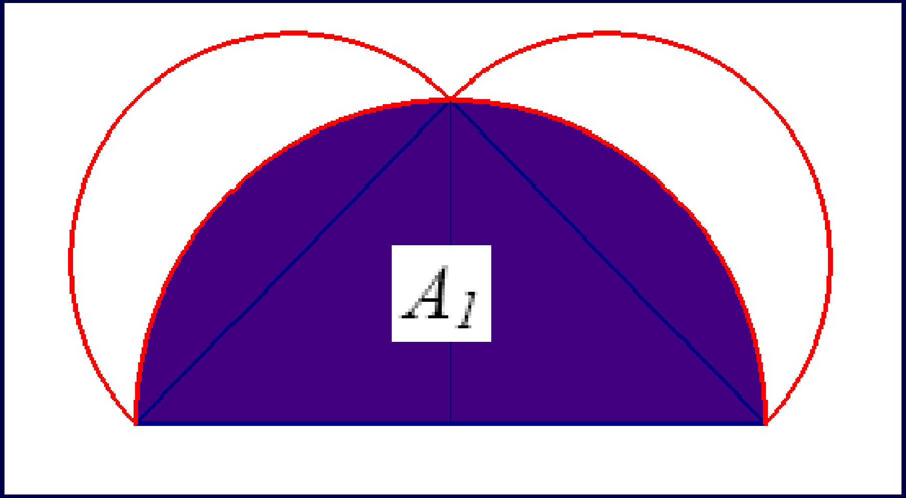


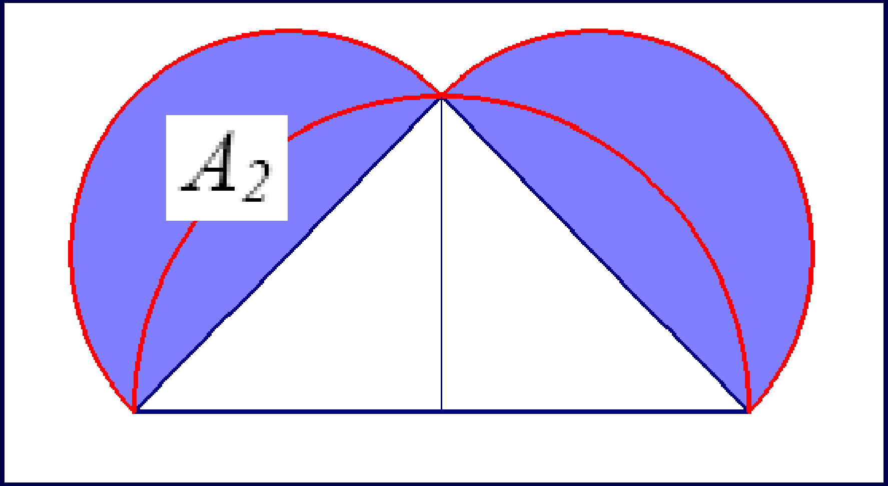


$$AB = AC\sqrt{2}$$

$$\frac{A_1}{A_2} = \frac{AB^2}{AC^2} = 2$$

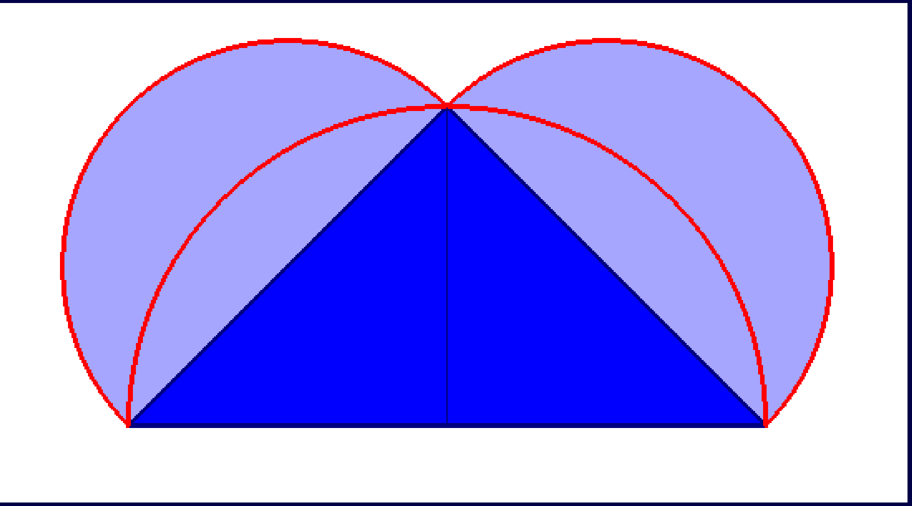
$$A_2 = 2\,A_1$$

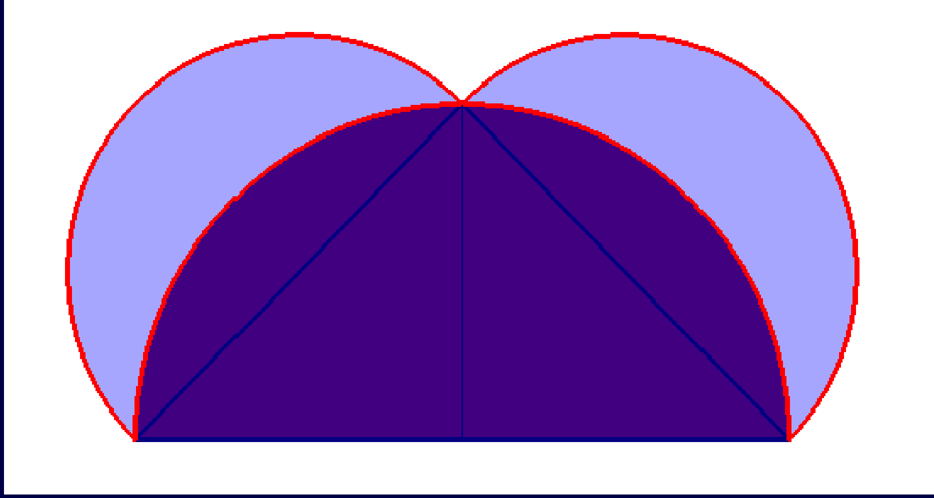


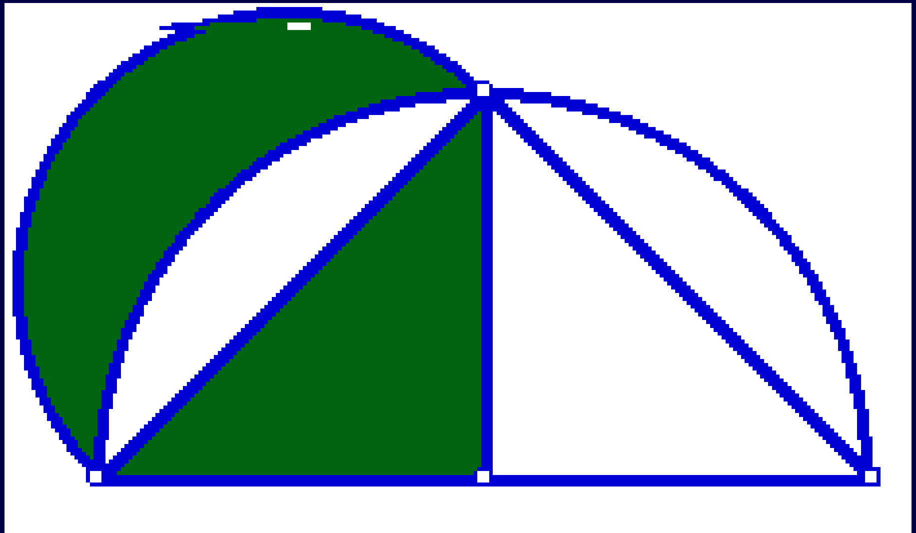


Hipócrates:

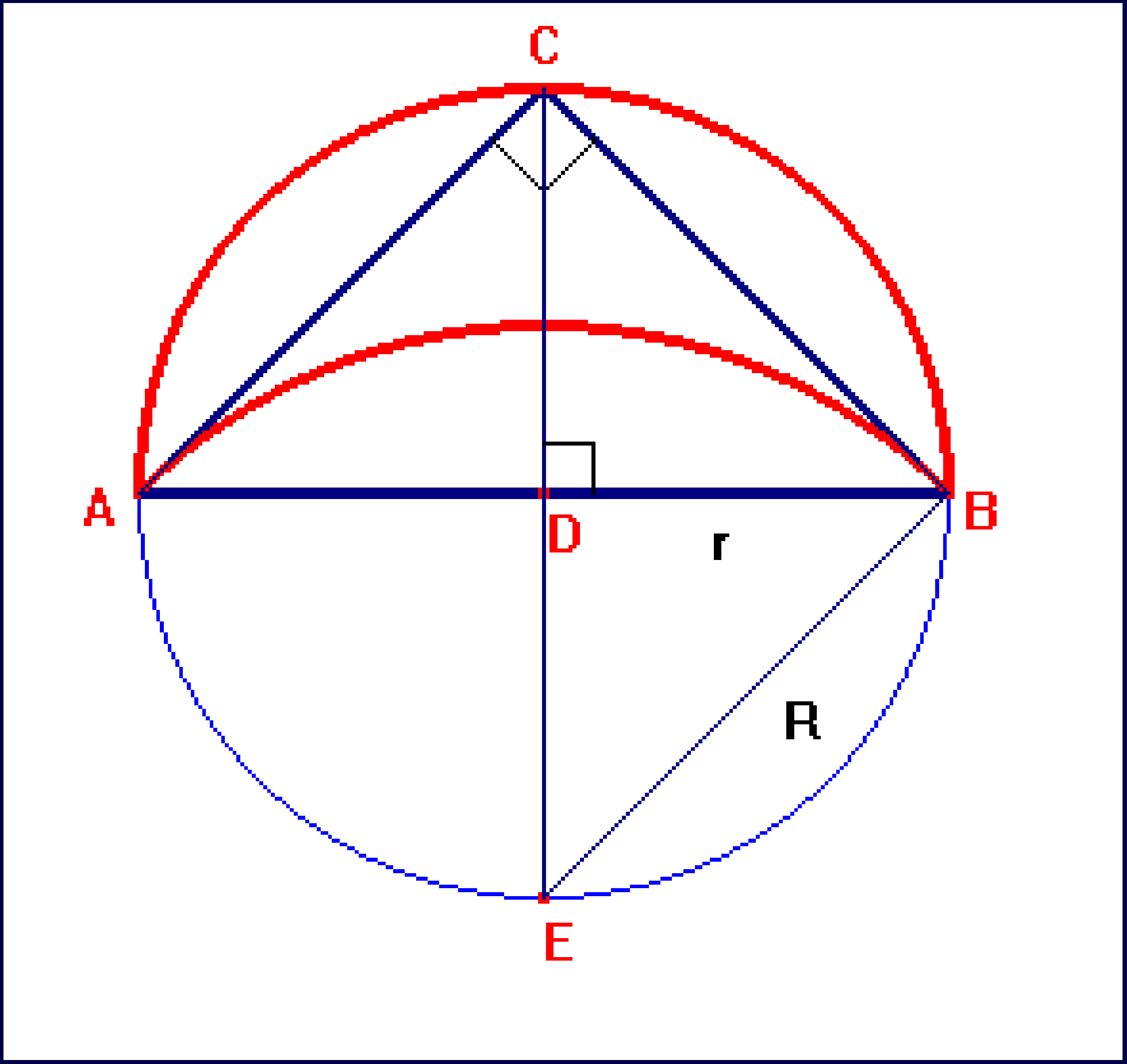
a área da lua é igual à área do triângulo

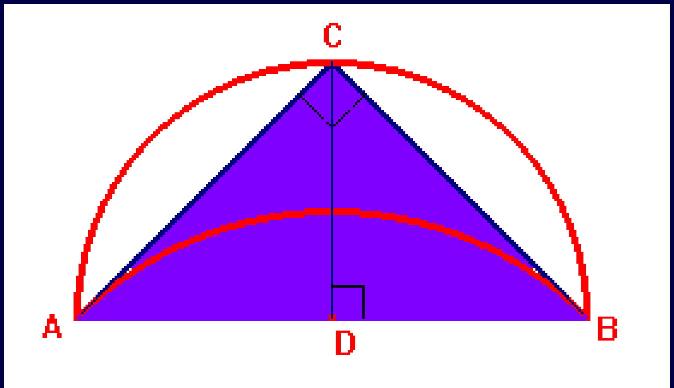


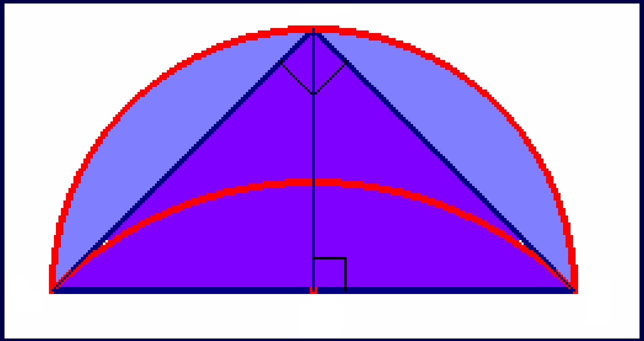


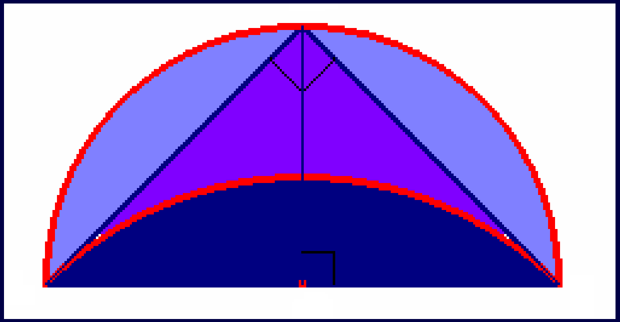


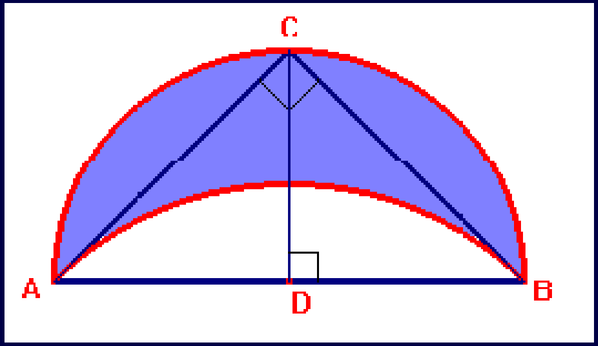
Versão alternativa...



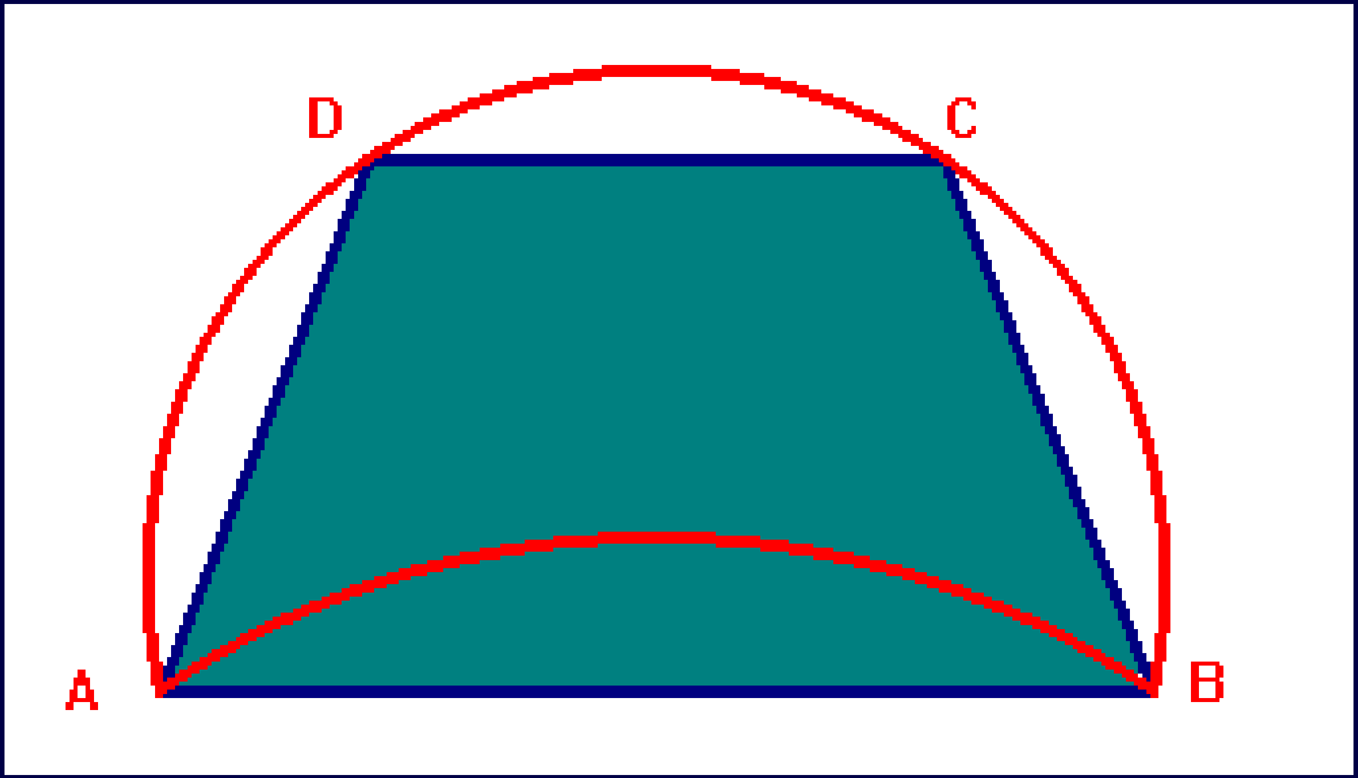

C
A
D
B

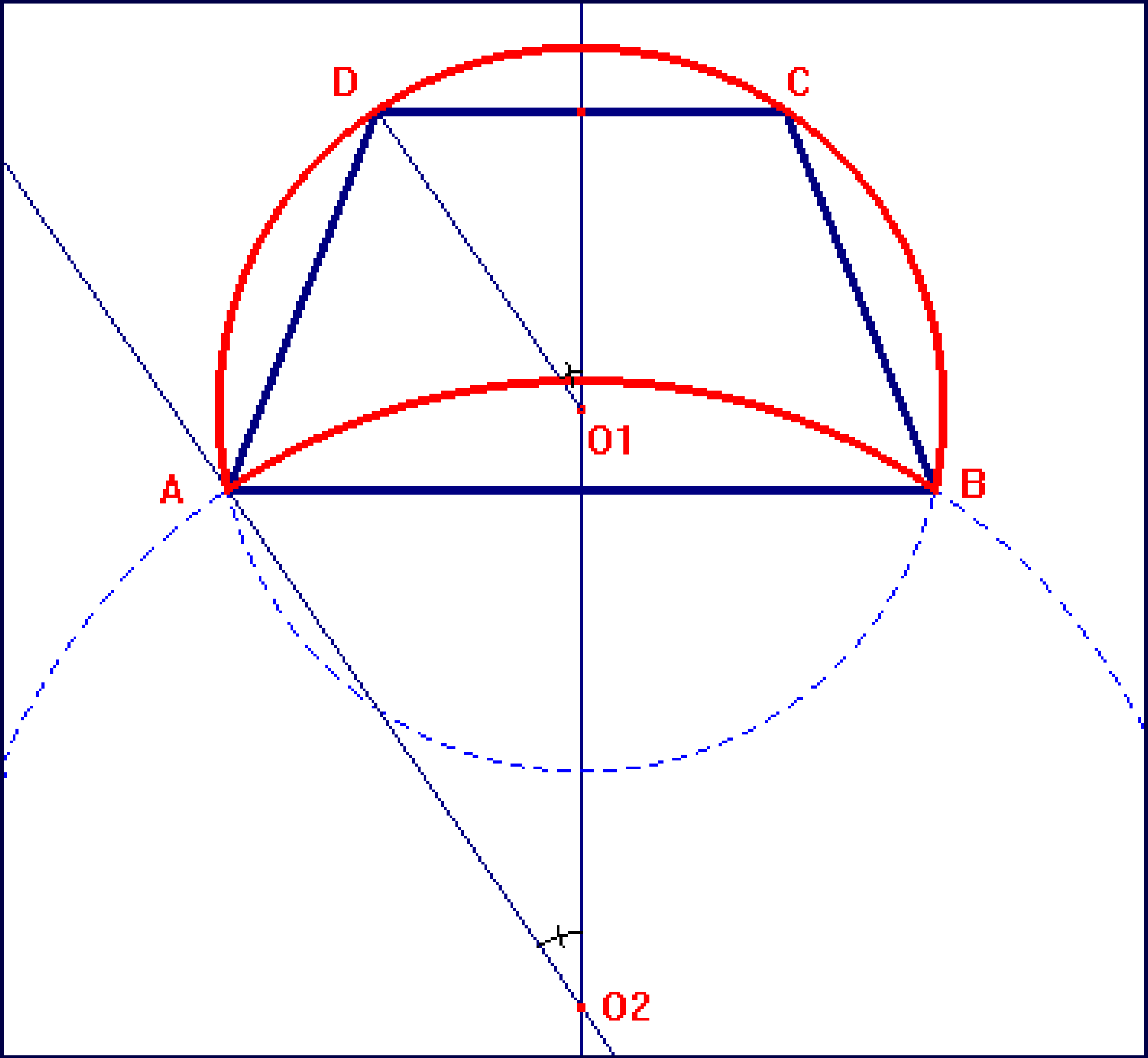





C
A
D
B

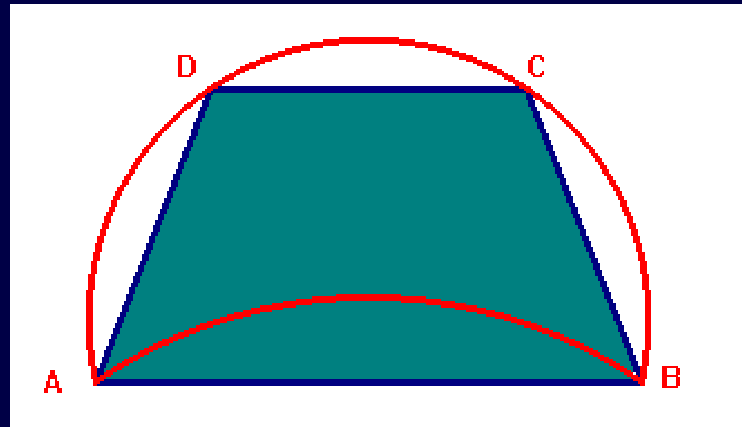
Segunda Lua de Hipócrates

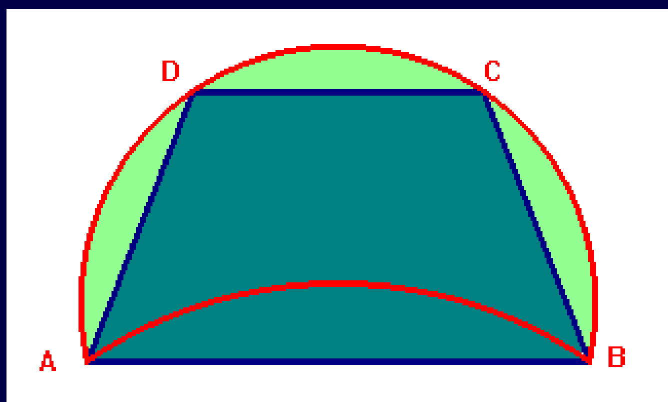


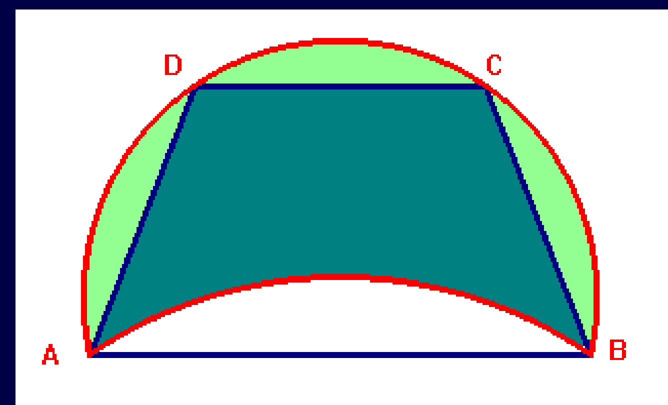


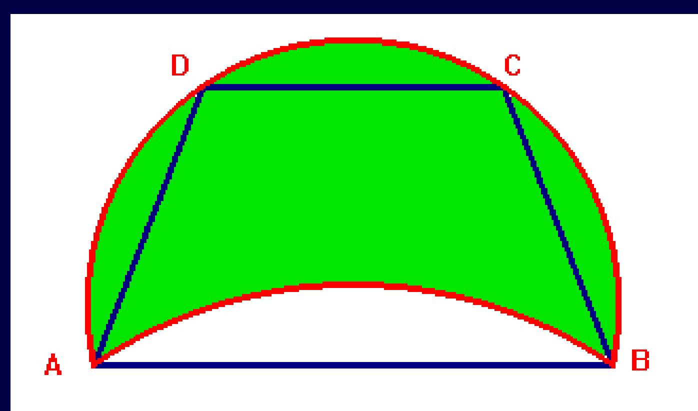
$AD = CD = BC = L$

$AB^2 = 3\ BC^2$

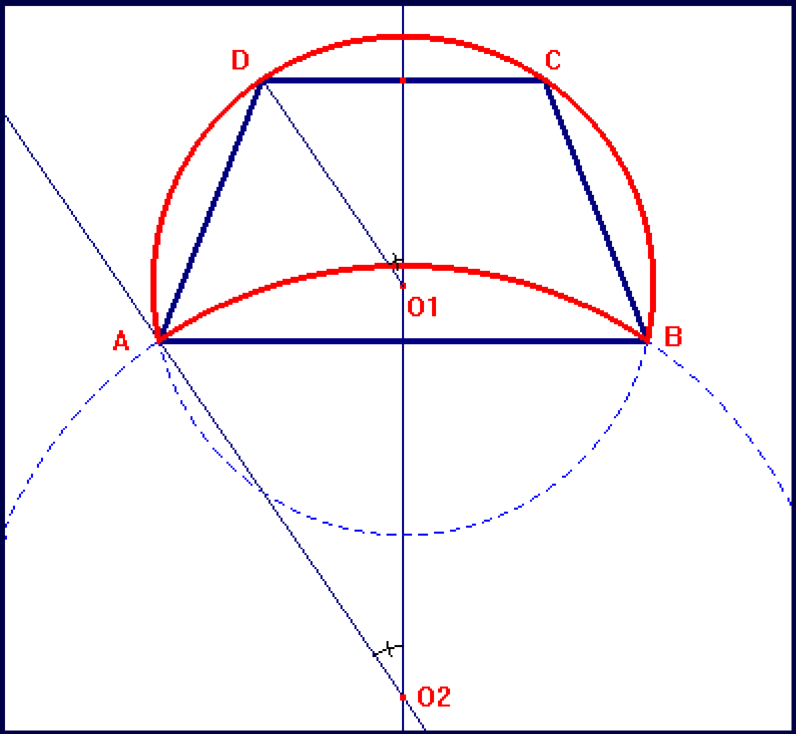

D
C
A
B


D
C
A
B


D
C
A
B


D
C
A
B

CONSTRUÇÃO DO TRAPÉZIO

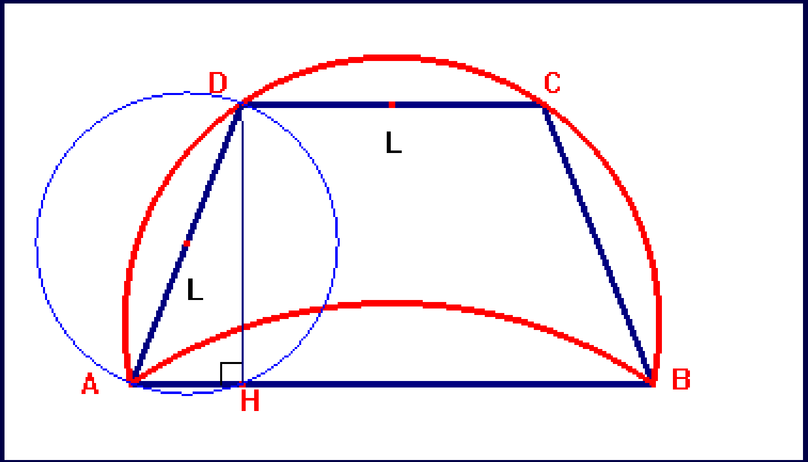


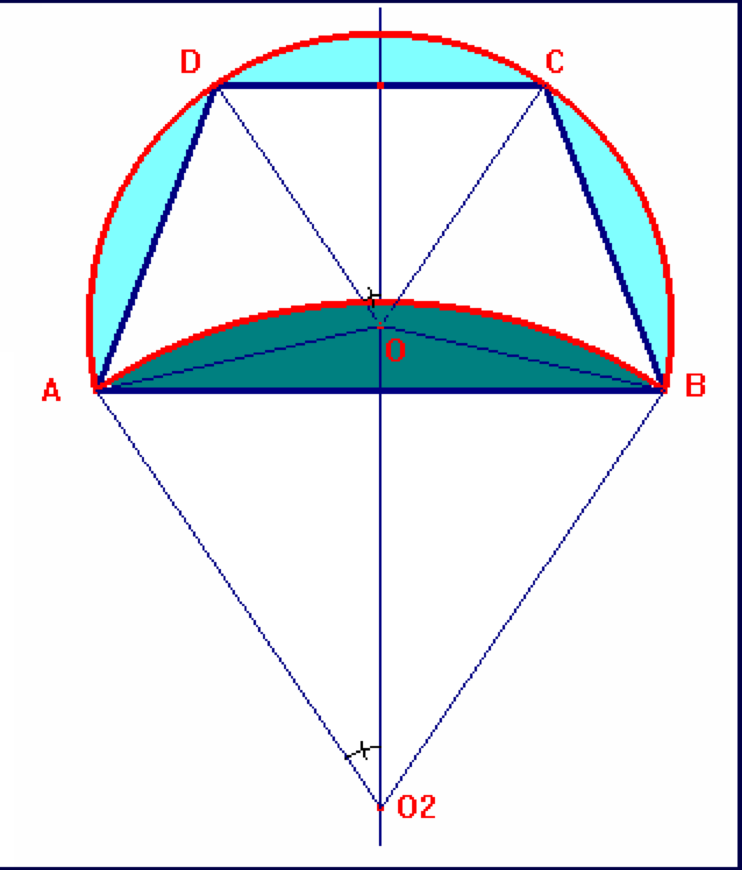
$$AB^2 = 3\ BC^2$$

ΔAHD triângulo retângulo

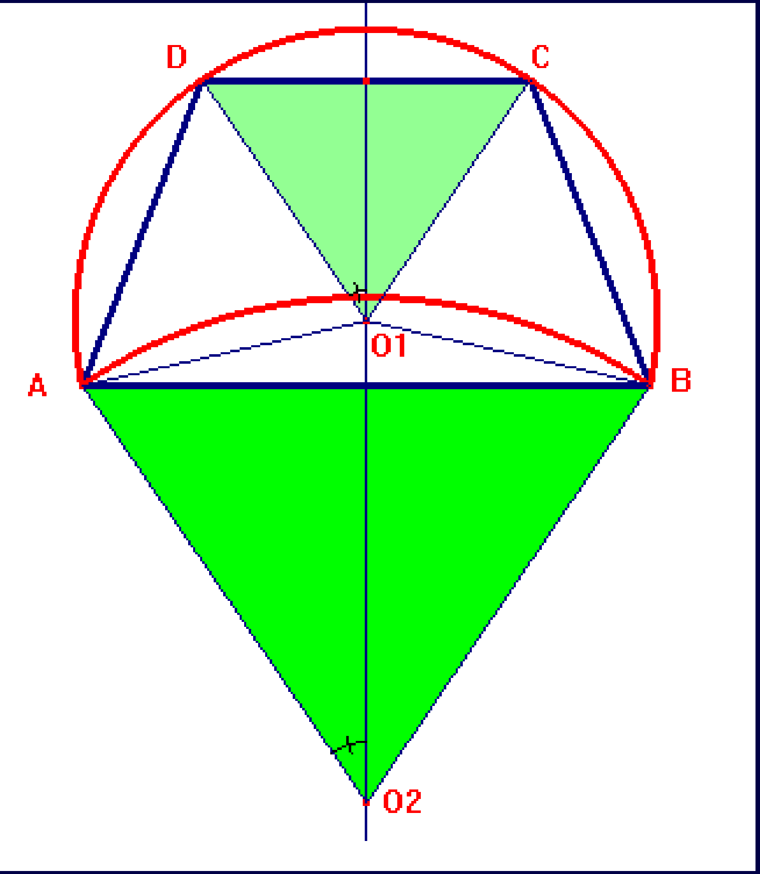
$AD = L$

$\overline{AH}$ mede $\dfrac{(\sqrt{3}-1)}{2}L$

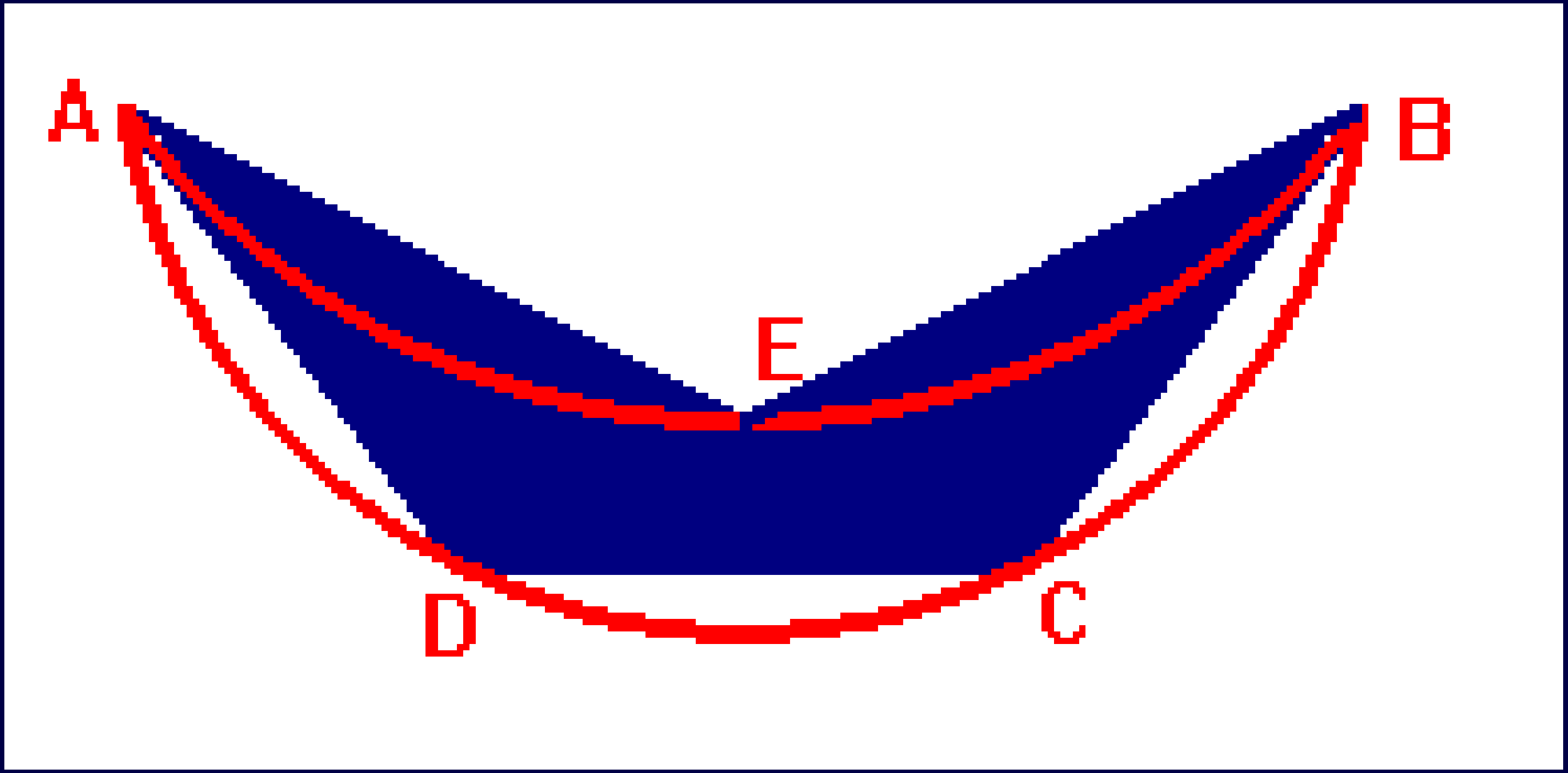


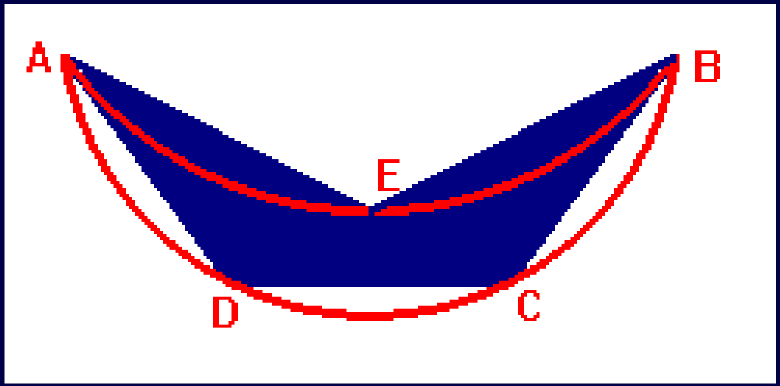


$$AB^2 = 3\ BC^2$$

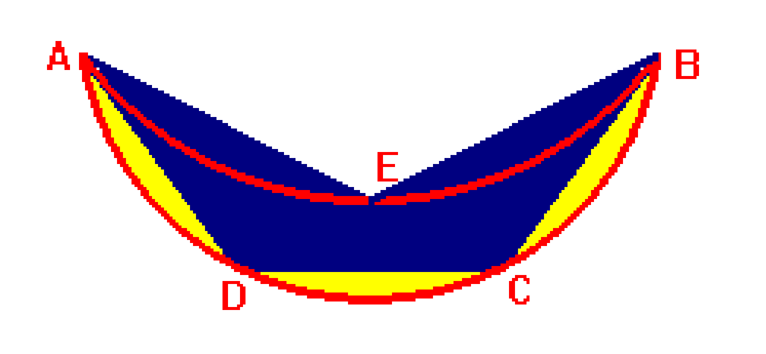


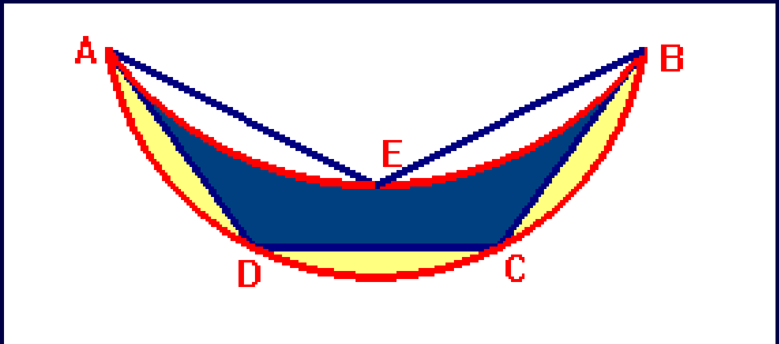
Terceira Lua de Hipócrates

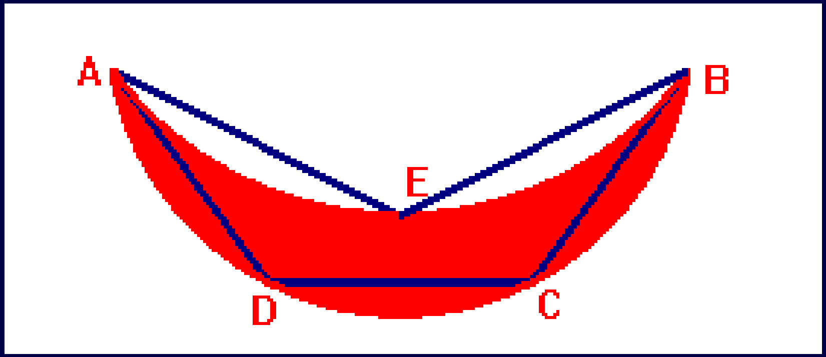




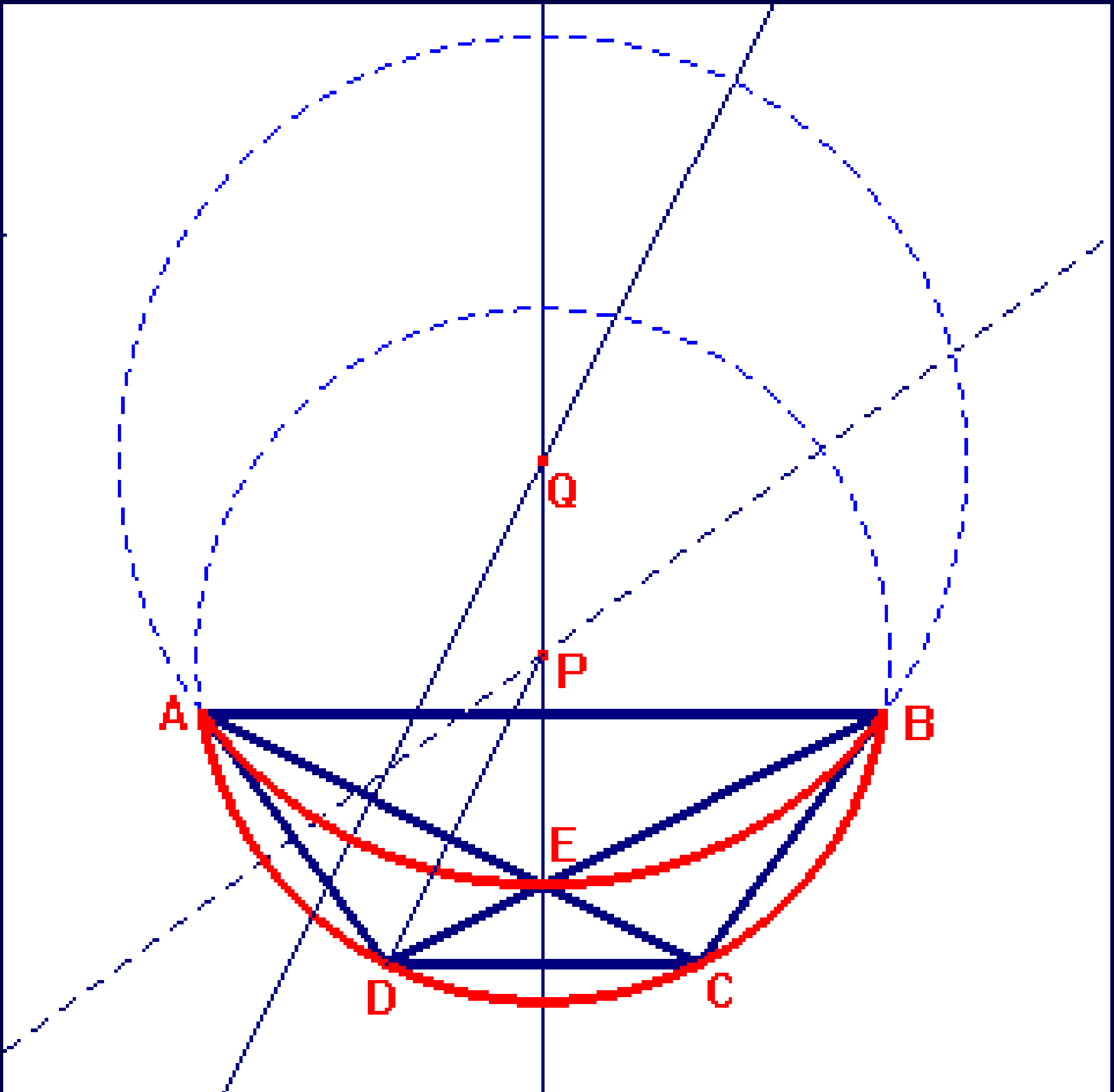
$$AE^2 = \frac{3}{2} BC^2$$

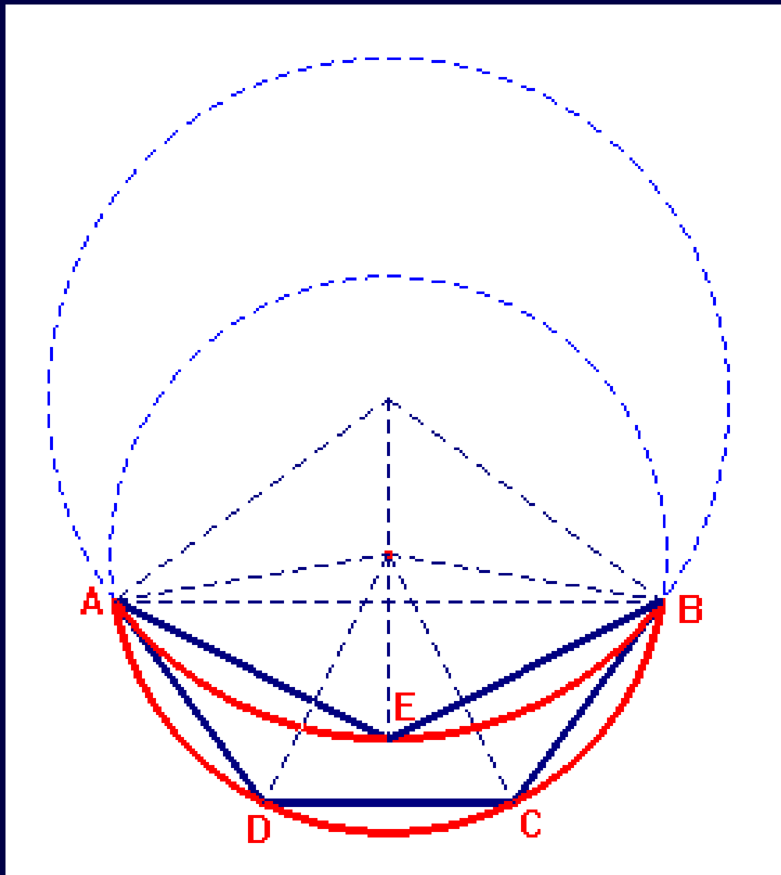




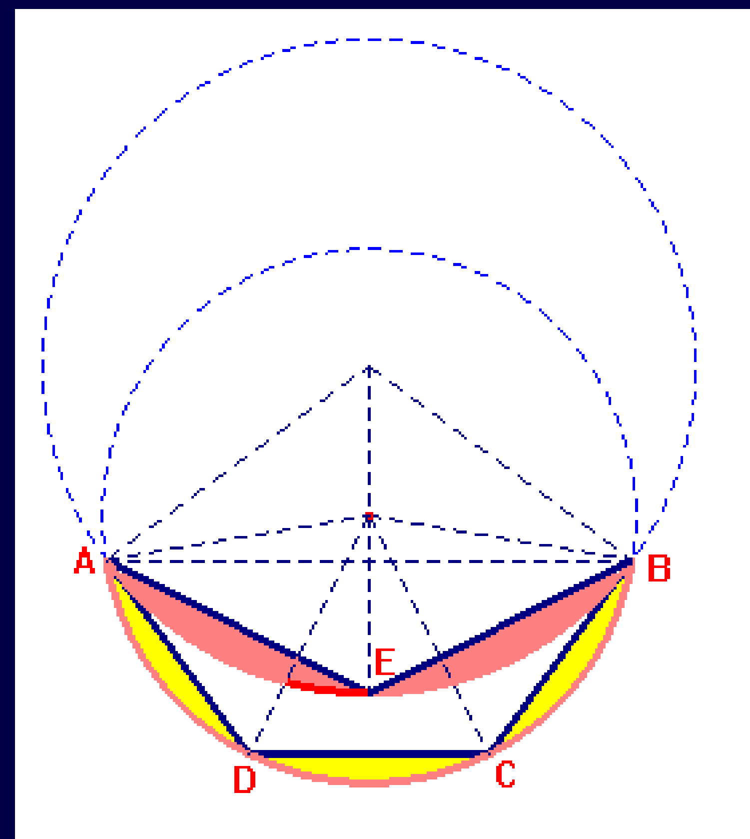


$$AE^2 = \frac{3}{2} BC^2$$



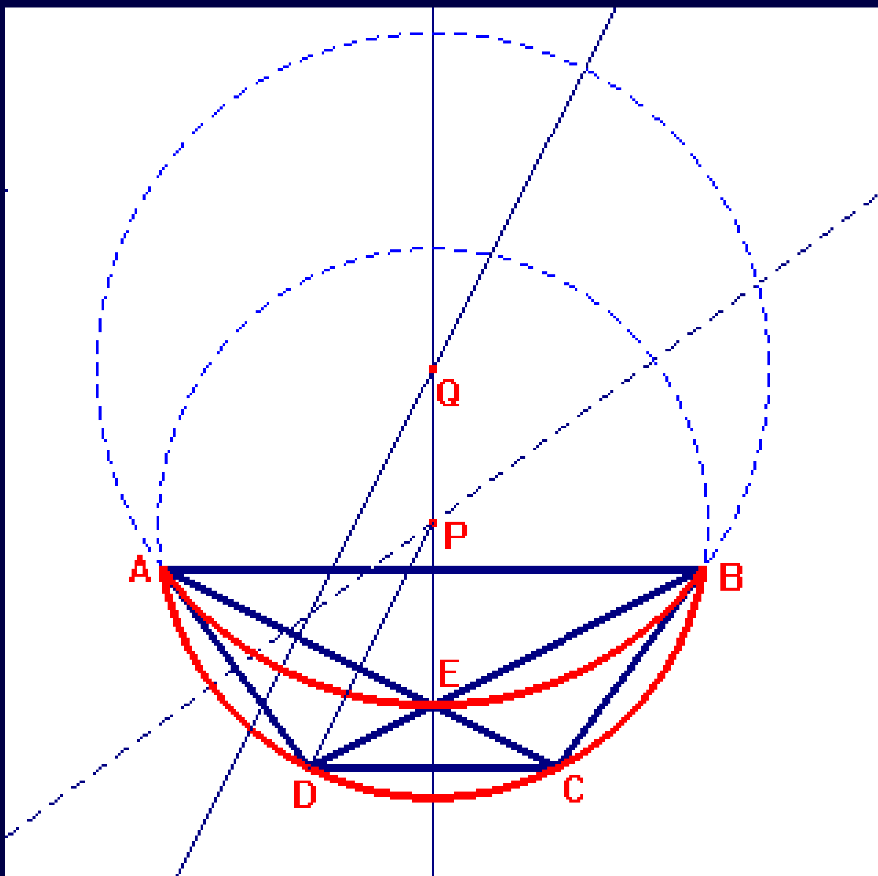


$$AE^2 = \frac{3}{2} BC^2$$



CONSTRUÇÃO DO POLÍGONO

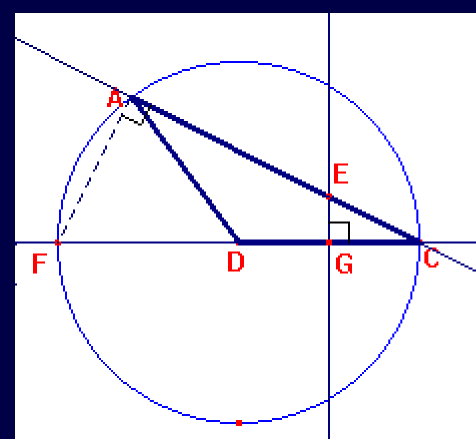
O triângulo ΔADC pode ser construído a partir de ***a* = *DC***



$$AC \,.\, EC = a^2$$

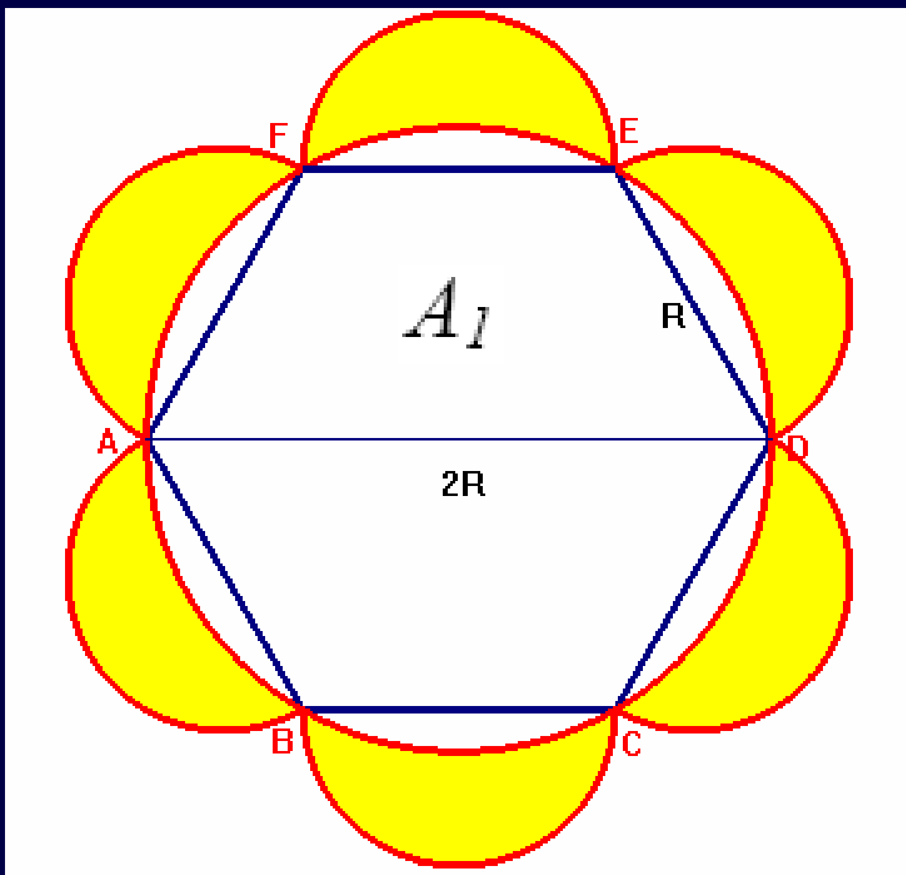
$$y = AC$$

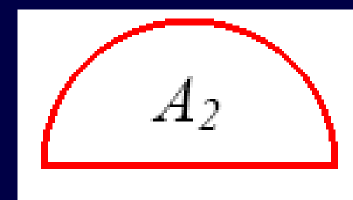
$$EC = y - \frac{\sqrt{3}}{\sqrt{2}}\,a$$



$$y = \frac{\sqrt{6}+\sqrt{22}}{4}\,a$$

HIPÓCRATES E A QUADRATURA

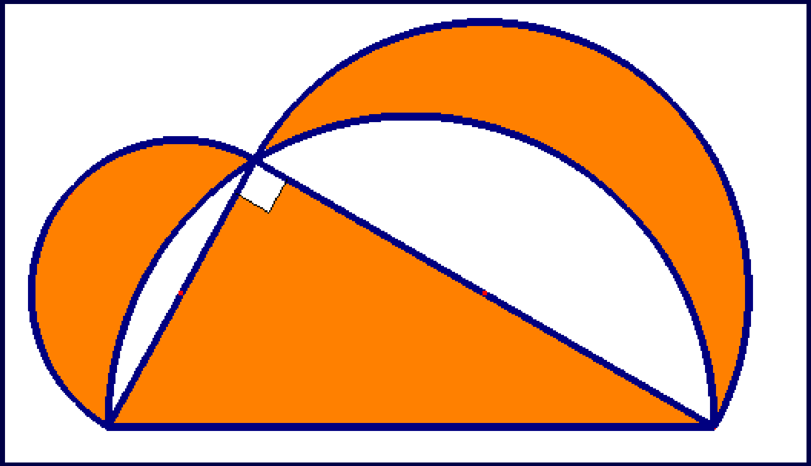




$$6\,A = A_H + 6\,A_2 - 2\,A_1;$$

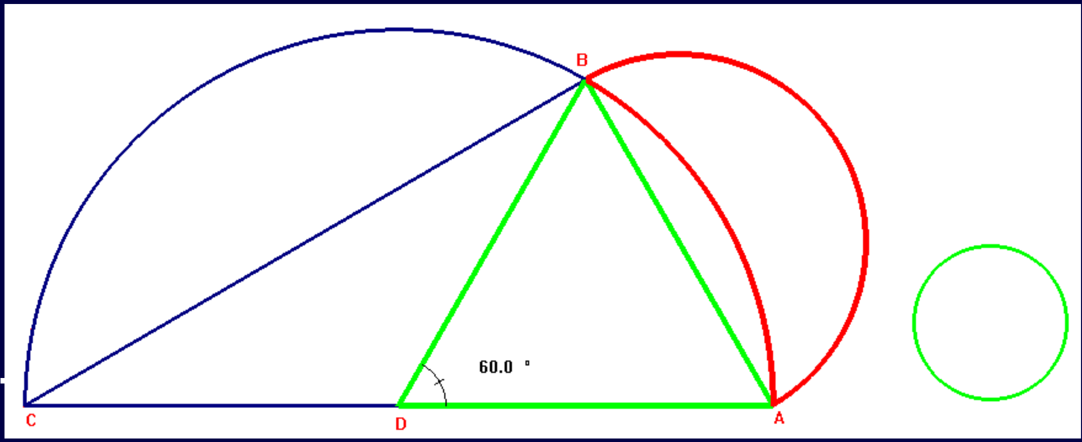
$$A_1 = 4\,A_2$$

Ibn Al-Haytham (965-1040)

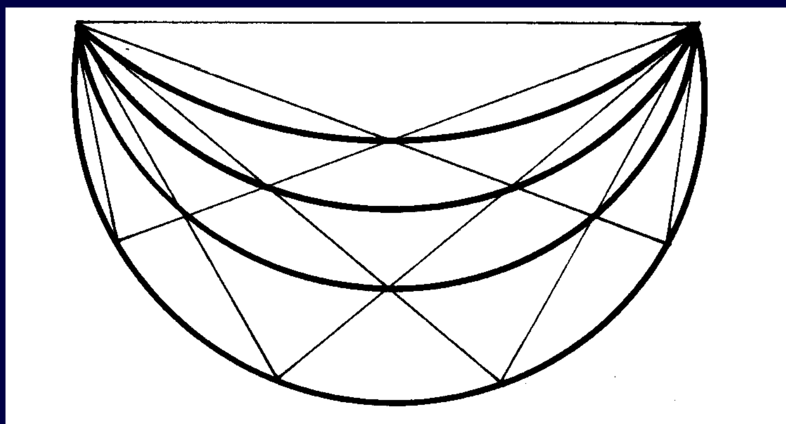


A reunião das luas é equivalente ao triângulo retângulo

A lua construída sobre o lado de um triângulo retângulo 30º - 60º é equivalente à reunião do triângulo equilátero cujo lado coincide com a corda AB e 1/24 do círculo cujo diâmetro é AB.



O PROBLEMA GERAL



$$\frac{S_m^2}{S_n^2} = \frac{n}{m} = \frac{A_m}{A_n}$$

De Lunarum Quadratura

Atribuída a Leon Battista Alberti
1404–1472

Trigonometria

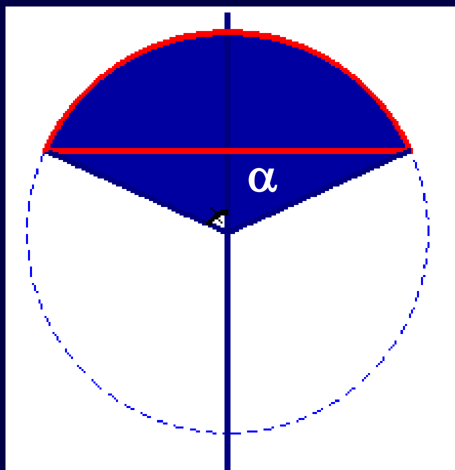
NOVAS SOLUÇÕES E CONTRIBUIÇÕES

Martin Johan Wallenius - 1766
Leonhard Euler – 1771
Thomas Clausen - 1840

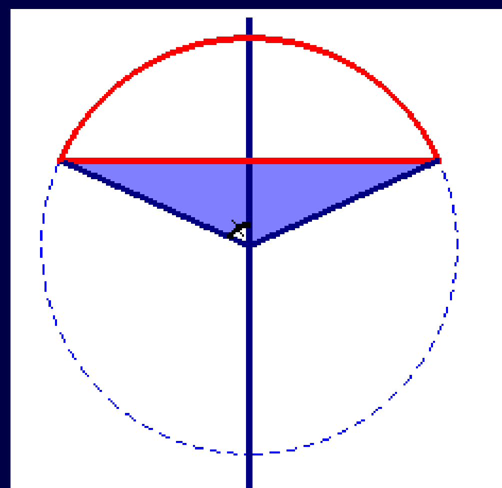
$$\frac{S^2_m}{S_n^2} = \frac{n}{m} = \frac{A_m}{A_n}$$

Novos exemplos
m, n : 5, 1 ou 5, 3, respectivamente

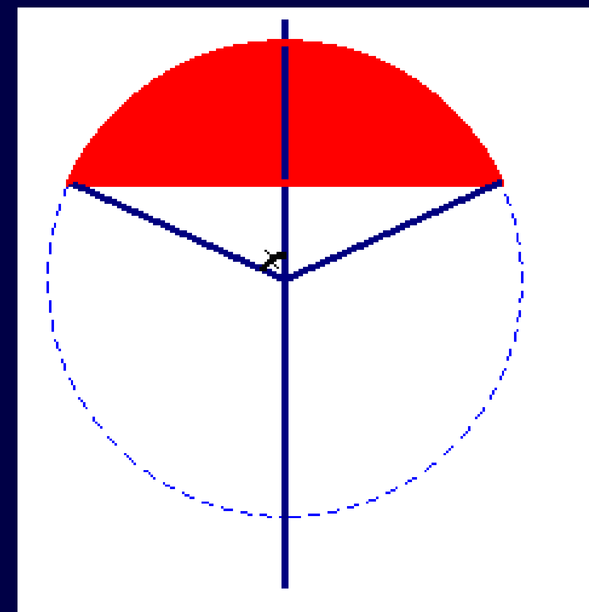
Calculando a área de uma lua
I. Segmentos Circulares



$$A_1 = r^2 \alpha$$



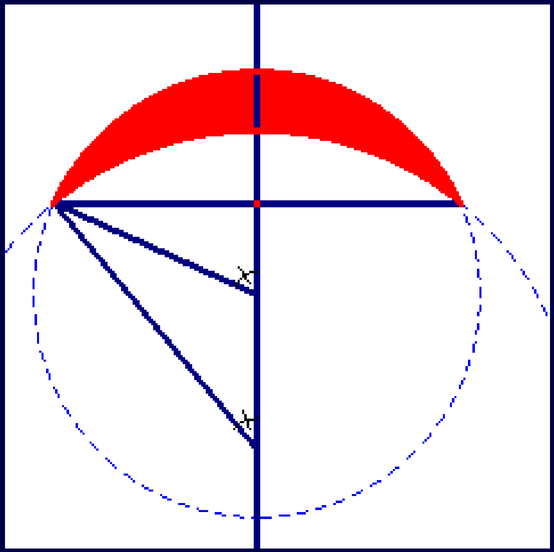
$$A_2 = \frac{1}{2} r^2 \text{sen } 2\alpha$$

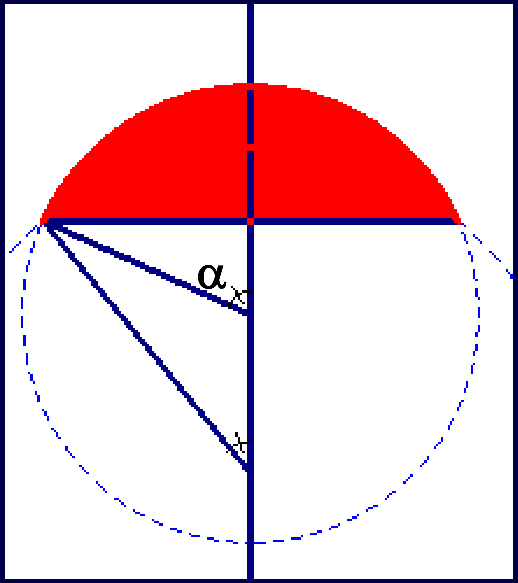


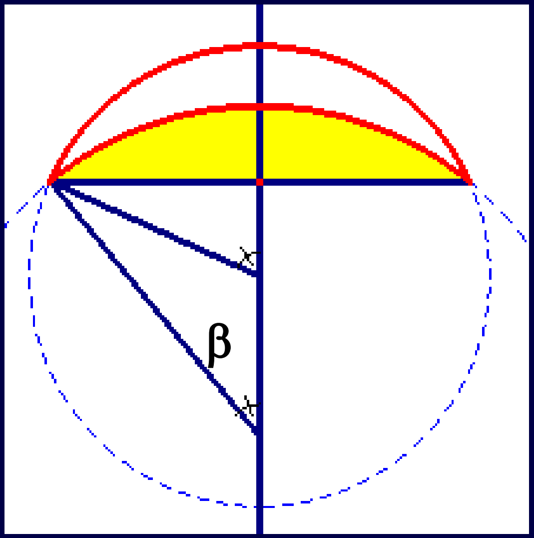
$$A = A_1 - A_2$$

$$= r^2 \alpha - \frac{1}{2} r^2 \text{sen } 2\alpha$$

Calculando a área de uma lua
II. Diferença de Áreas de Segmentos





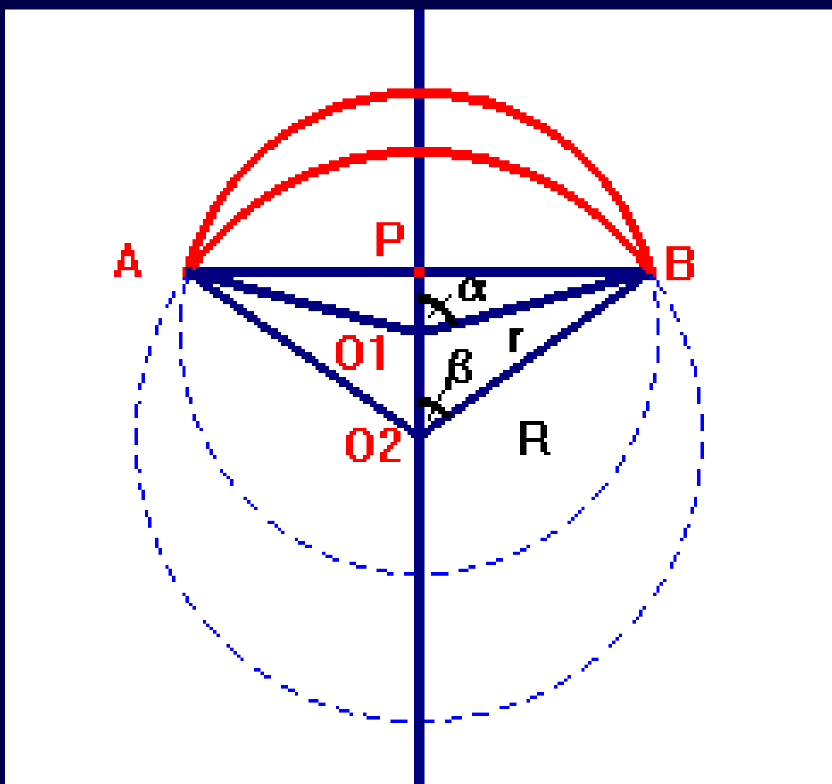


$$A = r^2\alpha - R^2\beta + \frac{1}{2}\,R^2 \mathrm{sen}\, 2\beta - \frac{1}{2}\,r^2 \mathrm{sen}\, 2\alpha.$$

$$A = r^2 \alpha - R^2 \beta + \frac{1}{2} R^2 \operatorname{sen} 2\beta - \frac{1}{2} r^2 \operatorname{sen} 2\alpha.$$

$$r^2 \alpha - R^2 \beta = 0$$

$$u = \frac{\alpha}{\beta}$$



$$R^2 = u\, r^2$$

$$PB = r \operatorname{sen} \alpha = R \operatorname{sen} \beta$$

$$\operatorname{sen}(u\,\beta) = \sqrt{u}\, \operatorname{sen}(\beta)$$

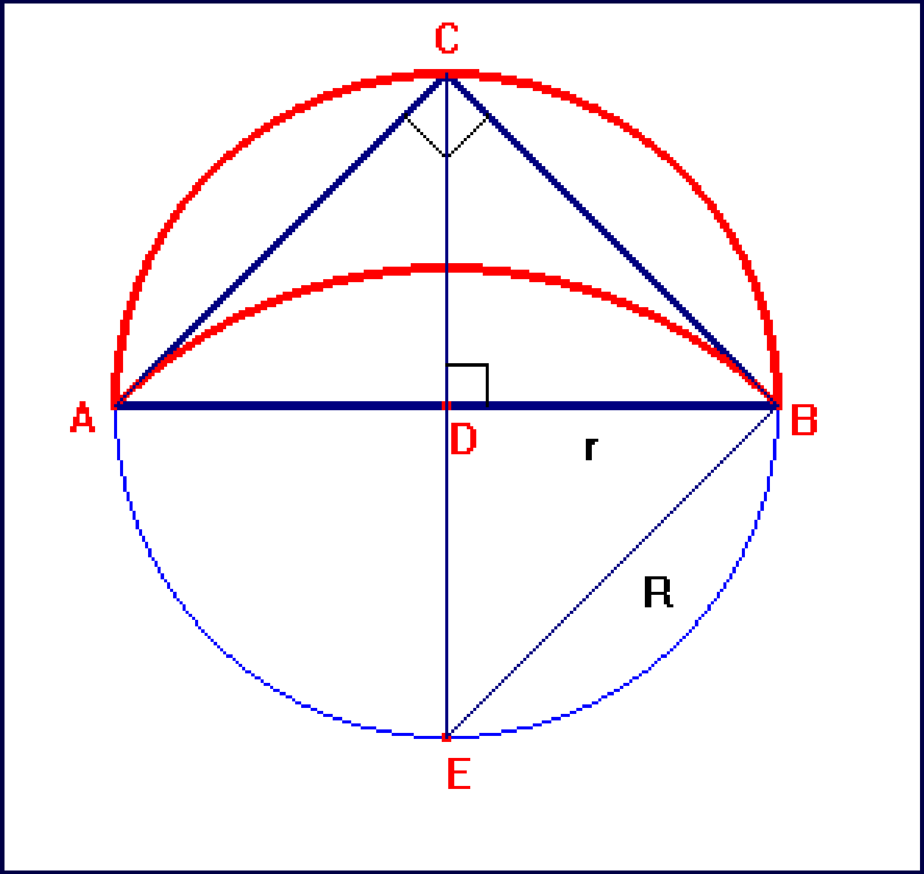
1^o caso: $u = 2$

$$R = r\sqrt{2}$$

$$\alpha = 2\beta$$

$$\sqrt{2}\,\text{sen}\beta = \text{sen}(2\beta)$$

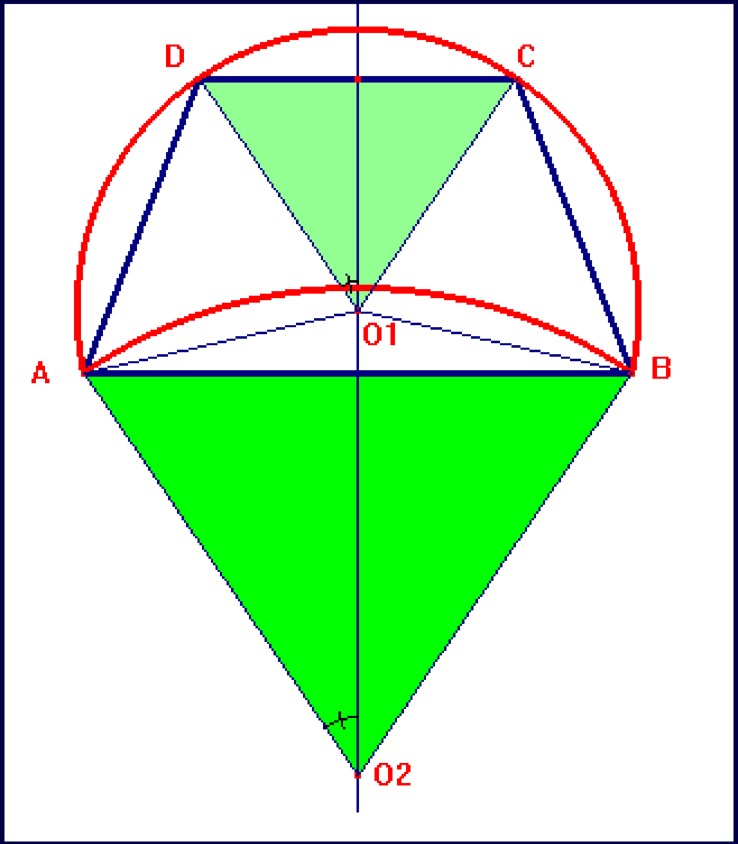
$$\beta = \pi/4 \text{ e } \alpha = \pi/2$$



2^o *caso: u = 3*

$$R = r\sqrt{3}$$

$$\alpha = 3\beta$$



$$\sqrt{3}\,\mathrm{sen}\beta = \mathrm{sen}\,(3\beta)$$

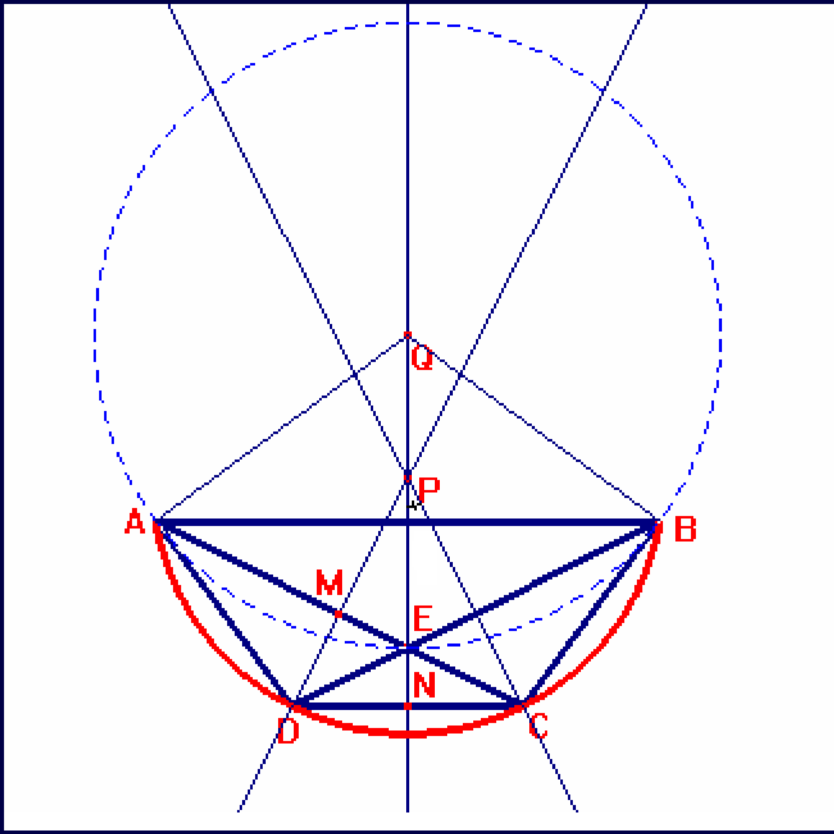
$$\sqrt{3}\,\mathrm{sen}\,\beta = 3\,\mathrm{sen}\,(\beta) - 4\,\mathrm{sen}^3(\beta)$$

$$\mathrm{sen}^2(\beta) = (3 - \sqrt{3})/4$$

3º caso: u = 3/2

$$u = \frac{\alpha}{\beta} = \frac{3}{2}, \quad R = \sqrt{\frac{3}{2}}\,r$$

$$\theta = \frac{\beta}{2}$$



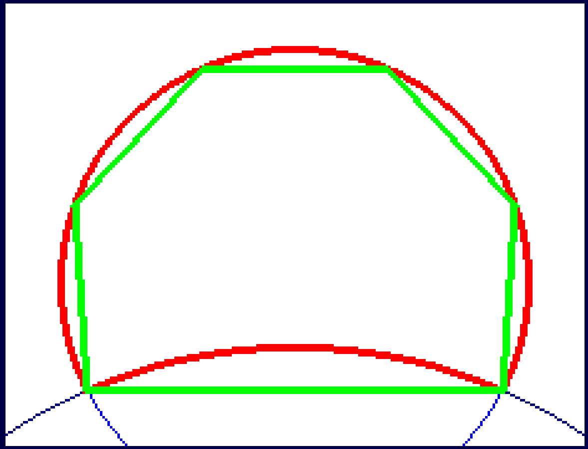
$$\sqrt{3}\,\text{sen}\,2\theta = \sqrt{2}\;\text{sen}\,3\theta$$

$$2\sqrt{3}\,\text{sen}\,\theta\,\cos\theta = \sqrt{2}\;\text{sen}\,\theta(3 - 4\,\text{sen}^2\,\theta)$$

$$\text{sen}\,\theta = \frac{\sqrt{9-\sqrt{33}}}{4}$$

4º caso: u = 5

$$u = \frac{\alpha}{\beta} = 5\ ,\ R = \sqrt{5}r$$



$$\sqrt{5}\ \text{sen}\ \beta = \text{sen}\ 5\beta$$

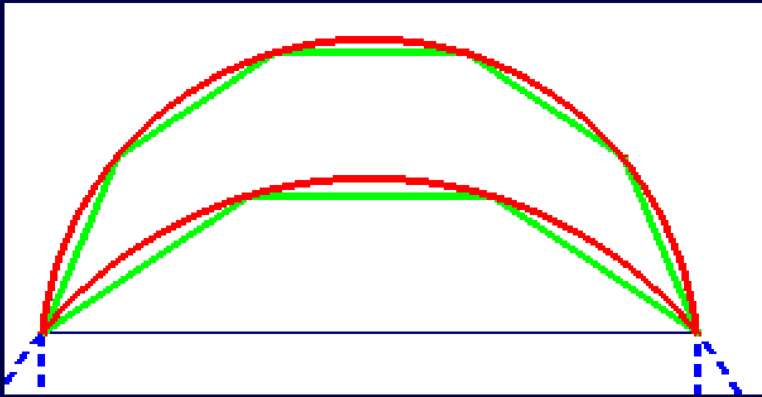
$$\sqrt{5}\ \text{sen}\ \beta = 5\ \text{sen}\ \beta - 20\ \text{sen}^3\beta + 16\ \text{sen}^5\beta$$

$$sen^2\beta \qquad \frac{5-\sqrt{4\sqrt{5}+5}}{8}$$

β é construtível e mede, aproximadamente, 23,5º

5º caso: u = 5/3

$$u = \frac{\alpha}{\beta} = \frac{5}{3}, \quad R = \sqrt{\frac{5}{3}}r$$



$$\sqrt{5}\ \text{sen}\, 3\theta = \sqrt{3}\ \text{sen}\, 5\theta$$

$$3\theta = \beta$$

$$\sqrt{3}\ (5\ \text{sen}\ \theta - 20\ \text{sen}^3\theta + 16\ \text{sen}^5\theta) = \sqrt{5}\ (3\ \text{sen}\ \theta - 4\ \text{sen}^3\theta)$$

$$sen^2\theta \qquad \frac{1}{24}(15 - \sqrt{15} \pm \sqrt{60 + 6\sqrt{15}})$$

16,81° corresponde ao sinal negativo.

SOLUÇÕES GERAIS

Tschakallof (1928)
Tschebotaröw (1934)
Dorodonow (1947)

não existência de outros exemplos construtíveis

1966 - a demonstração de não existência foi concluída por A. Baker:

Girstmair, K., Hippocrates'Lunes and Transcendence (2003)

Baker-1966

$$\frac{R^2}{r^2} = \frac{\alpha}{\beta} = \frac{m}{n} \quad (*)$$

$$r \, sen \, \alpha = R \, sen \, \beta \qquad (**)$$

$$n(sen(m\gamma))^2 = m(sen(n\gamma))^2 \quad (***).$$

polinômios de graus $2m$ e $2n$.

...EQUAÇÕES...

Obrigada
БЛАГОДАРЮ!
СПАСИБО!

Referências

DORODNOV, A.V. 1947. O kruvykh lunochkakh kvadriruemykh pri pomoshchi tsirkulya i lineiki. In: *Dokl. Akad. Nauk. SSSP*, **58**. 965-968.

GIRSTMAIR, Kurt. 2003. Hippocrates´lunes and transcendence. In: Expo. Math, **21**. 179-183.

HEATH, Sir Thomas. 1982. *A History of Greek Mathematics*. Vol. I. NY: Dover Pub. Inc.

KNORR, Wilbur Richard. 1986. *The Ancient Tradition of Geometric Problems*. NY: Dover Pub. Inc.

POSTINOV, M. M. 2000. The Problem of Squarable Lunes. In: *Am. Mat. Monthly*. **107**. 645-651.

TSCHAKALOFF, Ljubomir. 1929. Beitrag zum Problem der quadrierbaren Kreisbogenzweiecke, In: *Math. Z*. **30**. 552-559.

TSCHEBOTAÖW, Nikolaj. 1934. Über quadrierbare Kreisbogenzweiecke. In: *Math.Z*. **39**. 161-175.

WALLENIUS, Martin Johan. 1766. *Dissertatio Gradualis: Lunulas Quasdam Circulares Quadrabilis*, translated by Ian Bruce. In: www.17centurymaths.com/contents/lunes.pdf

ZEUTHEN, Hieronymus Georg. 1902. *Histoire des mathématiques dans l'antiquité et le Moyen Âge*. Paris: Gauthier-Villars.